CLASS NOTES -CHAPTER 9

SSLC

Mathematics

Quadratic Equations

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CHAPTER -9

QUADRATIC EQUATIONS

- Standard form of Quadratic equations:

  $\mathbf{ax^2 + bx + c = 0}$

  (a,b,c Real numbers and $a \neq 0$)

- Pure quadratic equations: $ax^2 + c = 0$

- Adfected quadratic equations: $\mathbf{ax^2 + bx + c = 0}$

  (a,b,c Real numbers and a &b $\neq 0$)

- Methods to solve quadratic equations:
  - Factorisation Method
  - Completing the square method
  - Formula method
  - Graphical method

- The roots of quadratic equation $ax^2 + bx + c = 0$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The sum of the roots quadratic equation:

$$m + n = \frac{-b}{a}$$

- The product of the roots of quadratic equation:

$$mn = \frac{c}{a}$$

➢ If m' and n are the roots of quadratic equation, then the standard form of the equation is

$$\boxed{\mathbf{x^2 - (m + n)x + mn = 0}}$$

➢ The discriminant of the quadratic equation

$$\boxed{\Delta = \mathbf{b^2 - 4ac}}$$

Discriminant	Nature of roots
$\Delta = \mathbf{0}$	Real and equal
$\Delta > \mathbf{0}$	Real and distinct
$\Delta < \mathbf{0}$	No real roots (imaginary roots)

➢ The graph of the form $y = x^2$ is called:

Parabola

Exercise 9.1

1. Check whether the following are quadratic equations :

i. $x^2 - x = 0$

The highest degree of the variable is 2

$\therefore$ The given equation is a quadratic equation

ii. $x^2 = 8$

The highest degree of the variable is 2

$\therefore$ The given equation is a quadratic equation

iii. $x^2 + \frac{1}{2}x = 0$

$x^2 + \frac{1}{2}x = 0 \Rightarrow \frac{2x^2+x}{2} = 0 \Rightarrow 2x^2 + x = 0$ The highest degree of the variable is 2

$\therefore$ The given equation is a quadratic equation

iv. $3x - 10 = 0$

The highest degree of the variable is 1

$\therefore$ The given equation is not a quadratic equation.

v. $x^2 - \frac{29}{4}x + 5 = 0$

$x^2 - \frac{29}{4}x + 5 = 0 \Rightarrow \frac{4x^2 - 29x + 20}{4} = 0 \Rightarrow 4x^2 - 29x + 20 = 0$

The highest degree of the variable is 2

$\therefore$ The given equation is a quadratic equation.

vi. $5 - 6x = \frac{2}{5}x^2$

$5 - 6x = \frac{2}{5}x^2 \Rightarrow \frac{2}{5}x^2 + 6x - 5 = 0$

$\Rightarrow \frac{2x^2 + 30x - 25}{5} = 0 \Rightarrow 2x^2 + 30x - 25 = 0$

The highest degree of the variable is 2

$\therefore$ The given equation is a quadratic equation.

vii. $\sqrt{2}x^2 + 3x = 0$

The highest degree of the variable is 2

$\therefore$ The given equation is a quadratic equation.

viii. $\sqrt{3}x = \frac{22}{13}$

The highest degree of the variable is 1

$\therefore$ The given equation is not a quadratic equation.

ix. $x^3 - 10x + 74 = 0$

The highest degree of the variable is 3

$\therefore$ The given equation is not a quadratic equation.

x. $x^2 - y^2$

It has two variables.

$\therefore$ The given equation is not a quadratic equation.

2. Simplify the following equations and check whether they are quadratic equations.

i. $x(x + 6) = 0$

$x(x + 6) = 0$

$\Rightarrow x^2 + 6x = 0$ The highest degree of the variable is 2

$\therefore$ The given equation is a quadratic equation.

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ii. $(x-4)(2x-3) = 0$

$(x-4)(2x-3) = 0$

$\Rightarrow x(2x-3) - 4(2x-3) = 0$

$\Rightarrow 2x^2 - 3x - 8x + 12 = 0$

$\Rightarrow 2x^2 - 11x + 12 = 0$

It is of the form $ax^2 + bx + c = 0$

$\therefore$ It is a quadratic equation.

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iii. $(x+9)(x-9) = 0$

$(x+9)(x-9) = 0$

$\Rightarrow x^2 - 9^2 = 0$

$\Rightarrow x^2 - 81 = 0$

It is of the form $ax^2 + bx + c = 0$

$\therefore$ It is a quadratic equation.

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iv. $(x+2)(x-7) = 5$

Sol: $(x+2)(x-7) = 5$

$\Rightarrow x(x-7) + 2(x-7) = 5$

$\Rightarrow x^2 - 7x + 2x - 14 = 5$

$\Rightarrow x^2 - 5x - 14 - 5 = 0$

$\Rightarrow x^2 - 5x - 19 = 0$

It is of the form $ax^2 + bx + c = 0$

$\therefore$ It is a quadratic equation.

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v. $3x + (2x-1)(x-9) = 0$

$3x + 2x(x-9) - 1(x-9)$

$\Longrightarrow 3x + 2x^2 - 18x - x + 9$

$\Longrightarrow 2x^2 - 16x + 9 = 0$

It is of the form $ax^2 + bx + c = 0$

$\therefore$ It is a quadratic equation.

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vi. $(x+1)^2 = 2(x-3)$

$(x+1)^2 = 2(x-3)$

$\Longrightarrow x^2 + 2x + 1 = 2x - 6$

$\Longrightarrow x^2 + 2x + 1 - 2x + 6 = 0$
$\Longrightarrow x^2 + 7 = 0$
It is of the form $ax^2 + bx + c = 0$
$\therefore$ It is a quadratic equation.

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vii. $(2x - 1)(x - 3) = (x + 5)(x - 1)$
$(2x - 1)(x - 3) = (x + 5)(x - 1)$
$\Rightarrow 2x(x - 3) - 1(x - 3) = x(x - 1) + 5(x - 1)$
$\Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$
$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$
$\Rightarrow 2x^2 - 7x + 3 - x^2 - 4x + 5 = 0$
$\Rightarrow x^2 - 11x + 8 = 0$
It is of the form $ax^2 + bx + c = 0$
$\therefore$ It is a quadratic equation.

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viii. $x^2 + 3x + 1 = (x - 2)^2$
$x^2 + 3x + 1 = (x - 2)^2$
$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$
$\Rightarrow 3x + 1 = -4x + 4$
$\Rightarrow 3x + 1 + 4x - 4 = 0$
$\Rightarrow 7x - 3 = 0$
The highest degree of the variable is 1
$\therefore$ The given equation is not a quadratic equation.
$(x + 2)^3 = 2x(x^2 - 1)$
$(x + 2)^3 = 2x(x^2 - 1)$
$\Rightarrow x^3 + 3(x)(2)(x + 2) + 2^3 = 2x^3 - 2x$
$\Rightarrow x^3 + 6x(x + 2) + 8 = 2x^3 - 2x$
$\Rightarrow x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$
$\Rightarrow 2x^3 - 2x - x^3 - 6x^2 - 12x - 8 = 0$
$\Rightarrow x^3 - 6x^2 - 14x - 8 = 0$
The highest degree of the variable is 3
$\therefore$ The given equation is not a quadratic equation.

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ix. $x^3 - 4x^2 - x + 1 = (x - 2)^3$
$x^3 - 4x^2 - x + 1 = (x - 2)^3$
$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 3(x)(2)(x - 2) - 2^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 6x(x-2) - 8$
$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$
$\Rightarrow -4x^2 - x + 1 = -6x^2 + 12x - 8$
$\Rightarrow -4x^2 - x + 1 + 6x^2 - 12x + 8 = 0$
$\Rightarrow 2x^2 - 13x + 9 = 0$
It is of the form $ax^2 + bx + c = 0$
$\therefore$ It is a quadratic equation.

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2. Represent the following in the form quadratic equations.

i. The product of two consecutive integers is 306

$x(x+1) = 306$
$\Longrightarrow x^2 + x = 306$
$\Longrightarrow x^2 + x - 306 = 0$

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ii. The length of a rectangular park (in metres) is one more than twice its breadth and its area is 528 m^2

breadth= x m length=$2x + 1$ m
$(2x+1)x = 528$
$\Longrightarrow 2x^2 + x - 528 = 0$

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iii. A train travels a distance of 480 km at uniform speed. If the speed had been 8 km/hr, then it would have taken 3 hour more to cover the same distance.

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$\frac{480}{x-8} - \frac{480}{x} = 3 \Longrightarrow \frac{480x - 480(x-8)}{x(x-8)} =$
$\Longrightarrow \frac{480x - 480x + 3840}{x^2 - 8x} = 3$
$\Longrightarrow 3840 = 3(x^2 - 8x)$
$\Longrightarrow 840 = 3x^2 - 24x$
$\Longrightarrow 3x^2 - 24x - 3840 = 0$

Excercise 9.2

1. Classify the following equations into pure and adfected quadratic equations

Sl.No	Quadratic equations	Type	Reason
1	$x^2 = 100$	pure	Variable second degree only
2	$x^2 + 6 = 6$	pure	$x^2 = 0$ Variable second degree only
3	$p(p - 3) = 1$	adfected	$p^2 - 3p = 1$ Variable in both second and first degree
4	$x^2 + 3 = 2x$	adfected	$x^2 - 2x + 3 = 0$ Variable in both second and first degree
5	$(x + 9)(x - 9) = 0$	pure	$x^2 - 81$ Variable second degree only
6	$2x^2 = 72$	pure	Variable second degree only
7	$x^2 - x = 0$	adfected	Variable in both second and first degree
8	$7x = \frac{35}{x}$	pure	$7x^2 = 35$ Variable second degree only
9	$x + \frac{1}{x} = 5$	adfected	$x^2 - 5x + 1 = 0$ Variable in both second and first degree
10	$4x = \frac{81}{x}$	pure	$4x^2 = 81$ Variable second degree only
11	$(2x - 5)^2 = 81$	adfected	$4x^2 - 20x - 56 = 0$ Variable in both second and first degree
12	$\frac{(x - 4)^2}{18} = \frac{2}{9}$	adfected	$x^2 - 8x + 12 = 0$Variable in both second and first degree

2. Solve the quadratic equations :

i. $x^2 - 196 = 0$

$x^2 - 196 = 0$

$\Rightarrow x^2 = 196$

$\Rightarrow x = \sqrt{196} = \pm 14$

$\Rightarrow x = 15 \text{ or } x = -15$

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ii. $5x^2 = 625$

$5x^2 = 625$

$\Rightarrow x^2 = \dfrac{625}{5}$

$\Rightarrow x^2 = 125$

$\Rightarrow x = \sqrt{125} = \sqrt{25 \times 5} = \pm 5\sqrt{5}$

$\Rightarrow x = 5\sqrt{5} \text{or } x = -5\sqrt{5}$

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iii. $x^2 + 1 = 101$

$x^2 + 1 = 101$

$\Rightarrow x^2 = 101 - 1$

$\Rightarrow x^2 = 100$

$\Rightarrow x = \sqrt{100} = \pm 10$

$\Rightarrow x = 10 \text{ or } x = -10$

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iv. $7x = \frac{64}{7x}$

$7x = \dfrac{64}{7x}$

$\Rightarrow 49x^2 = 64$

$\Rightarrow x^2 = \frac{64}{49}$

$\Rightarrow x = \sqrt{\dfrac{64}{49}} = \pm \dfrac{8}{7}$

$\Rightarrow x = \frac{8}{7} \text{ or } x = -\frac{8}{7}$

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v. $(x + 8)^2 - 5 = 31$

$(x + 8)^2 - 5 = 31$

$\Longrightarrow (x + 8)^2 = 31 + 5$

$\Longrightarrow (x + 8)^2 = 36$

$\Longrightarrow x + 8 = \sqrt{36}$

$\Longrightarrow x+8=\pm 6$
$\Longrightarrow x=\pm 6-8$
$\Longrightarrow x=6-8$ or $\Longrightarrow x=-6-8$
$\Longrightarrow x=-2$ or $\Longrightarrow x=-14$

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vi. $\frac{x^2}{2}-\frac{3}{4}=7\frac{1}{4}$

$$\frac{x^2}{2}-\frac{3}{4}=7\frac{1}{4}$$

$\Rightarrow \frac{x^2}{2}=7\frac{1}{4}+\frac{3}{4}$

$\Rightarrow \frac{x^2}{2}=8$

$\Rightarrow x^2=16$

$\Rightarrow x=\sqrt{16}=\pm 4$

$\Longrightarrow x=4$ or $\Longrightarrow x=-4$

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vii. $-4x^2+324=0$

$-4x^2+324=0$

$\Rightarrow -4x^2=-324$

$\Rightarrow x^2=\frac{-324}{-4}$

$\Rightarrow x^2=81$

$\Rightarrow x=\sqrt{81}=\pm 9$

$\Longrightarrow x=9$ or $\Longrightarrow x=-9$

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viii. $-37.5x^2=-37.5$

$-37.5x^2=-37.5$

$$\Rightarrow x^2=\frac{-37.5}{-37.5}$$

$\Rightarrow x^2=1$

$\Rightarrow x=\sqrt{1}=\pm 1$

$\Longrightarrow x=1$ or $\Longrightarrow x=-1$

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3. In each of the following , determine whether the given values of 'x' is a solution of quadratic equation or not.

i. $x^2+14x+13=0\,;x=-1\,,\ x=-13$

$x^2+14x+13=0$

$x=-1$

$\Rightarrow (-1)^2+14(-1)+13$

$= 1 - 14 + 13$
$= 14 - 14 = 0$
$x = -13$
$\Rightarrow (-13)^2 + 14(-13) + 13$
$= 169 - 182 + 13$
$= 182 - 182 = 0$
$\therefore\ -1$ and -13 are the solution of the quadratic equation
$x^2 + 14x + 13 = 0$

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ii. $7x^2 - 12x = 0\ ;\ x = \frac{1}{3}$
$7x^2 - 12x = 0$
$x = \frac{1}{3}$
$\Rightarrow\ 7\left(\frac{1}{3}\right)^2 - 12\left(\frac{1}{3}\right)$
$= \frac{7}{9} - 4$
$= \frac{7-36}{9} = \frac{-29}{9} \neq 0$
$\therefore\ \frac{1}{3}$ is not a solution of the quadratic equation
$7x^2 - 12x = 0$

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iii. $2m^2 - 6m + 3 = 0\ ;\ m = \frac{1}{2}$
$2m^2 - 6m + 3 = 0$
$m = \frac{1}{2}$
$\Rightarrow 2\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) + 3$
$= 2\left(\frac{1}{4}\right) - 3 + 3$
$= \frac{1}{2} \neq 0$
$\therefore\ \frac{1}{2}$ *is not a soln of the quadratic equation* $2m^2 - 6m + 3 = 0$

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iv. $y^2 + \sqrt{2}y - 4 = 0\ ;\ y = 2\sqrt{2}$
$y^2 + \sqrt{2}y - 4 = 0$
$y = 2\sqrt{2}$
$\Rightarrow\ \left(2\sqrt{2}\right)^2 + \sqrt{2}\left(2\sqrt{2}\right) - 4$

$= 4(2) + 2(2) - 4$
$= 8 + 4 - 4$
$= 8 \neq 0$
$\therefore\ 2\sqrt{2}$ *is not a soln of the quadratic eqn* $y^2 + \sqrt{2}y - 4 = 0$

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v. $\frac{2x+1}{x} = 3x\ ; x = 1, x = -1,$

$$\frac{2x+1}{x} = 3x$$

$$x = 1 \Rightarrow \frac{2(1)+1}{1} = 3(1)$$

$$\Rightarrow \frac{3}{1} = 3$$

$$\Rightarrow 3 = 3$$

$$x = -1 \Rightarrow \frac{2(-1)+1}{1} = 3(-1)$$

$$\Rightarrow \frac{-1}{1} = -3$$

$$\Rightarrow -1 \neq 3$$

$\therefore$ 1 is a solution of the quadratic equation $\frac{2x+1}{x} = 3x$

$\therefore -1$ is not a solution of the quadratic equation $\frac{2x+1}{x} = 3x$

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vi. $(3k+8)(2k+5) = 0; k = 2\frac{2}{3}, k = 2\frac{1}{2}$

$$(3k+8)(2k+5) = 0$$

$$k = 2\tfrac{2}{3} \Rightarrow \left[3\left(2\tfrac{2}{3}\right) + 8\right]\left[2\left(2\tfrac{2}{3}\right) + 5\right]$$

$$= \left[3\left(\frac{8}{3}\right) + 8\right]\left[2\left(\frac{8}{3}\right) + 5\right]$$

$$= [8+8]\left[\frac{16}{3} + 5\right]$$

$$= [16]\left[\frac{16+15}{3}\right]$$

$$= [16]\left[\frac{31}{3}\right]$$

$$= \frac{496}{3} \neq 0$$

$$k = 2\tfrac{1}{2} \Rightarrow \left[3\left(2\tfrac{1}{2}\right) + 8\right]\left[2\left(2\tfrac{1}{2}\right) + 5\right]$$

$$= \left[3\left(\frac{5}{2}\right)+8\right]\left[2\left(\frac{5}{2}\right)+5\right]$$

$$= \left[\frac{15}{2}+8\right][5+5]$$

$$= \left[\frac{15+16}{2}\right][10]$$

$$= \left[\frac{31}{2}\right][10]$$

$$= \frac{310}{2} = 155 \neq 0$$

$\therefore\ 2\frac{2}{3}$ and $2\frac{1}{2}$ are not the solution of the quadratic equation $(3k+8)(2k+5)=0$

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vii. $\frac{x}{x+2} = \frac{1}{2}$; $x = 2, x = 1$

$$\frac{x}{x+2} = \frac{1}{2}$$

$$x = 2 \Rightarrow \frac{2}{2+2} = \frac{1}{2}$$

$$\Rightarrow \frac{2}{4} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{2}$$

$$x = 1 \Rightarrow \frac{1}{1+2} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{3} \neq \frac{1}{2}$$

$\therefore$ 2 is a solution of the quadratic equation $\frac{x}{x+2} = \frac{1}{2}$

$\therefore$ 1 is not a solution of the quadratic equation $\frac{x}{x+2} = \frac{1}{2}$

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viii. $6x^2 - x - 2 = 0$; $x = -\frac{1}{2}, x = \frac{2}{3}$

Sol : $6x^2 - x - 2 = 0$

$$x = -\frac{1}{2} \Rightarrow 6\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 2$$

$$= 6\left(\frac{1}{4}\right) + \frac{1}{2} - 2$$

$$= \frac{3}{2} + \frac{1}{2} - 2$$

$$= \frac{3+1-4}{2} = \frac{0}{2} = 0$$

$x = \frac{2}{3} \Rightarrow 6\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right) - 2$

$= 6\left(\frac{4}{9}\right) - \frac{2}{3} - 2$

$= \frac{8}{3} - \frac{2}{3} - 2$

$= \frac{8-2-6}{3} = \frac{0}{3} = 0$

$\therefore\ -\frac{1}{2}$ and $\frac{2}{3}$ are the soln of the quadratic eqn

$6x^2 - x - 2 = 0$

4.

i. If $A = \pi r^2$, solve for r , and find the value of 'r' if $A = 77$ and $\pi = \frac{22}{7}$

$A = \pi r^2$

$\Longrightarrow \pi r^2 = A$

$\Longrightarrow r^2 = \frac{A}{\pi}$

$\Longrightarrow r = \pm\sqrt{\frac{A}{\pi}}$

$\Longrightarrow r = \pm\sqrt{\frac{77}{\frac{22}{7}}}$

$\Longrightarrow r = \pm\sqrt{77 \times \frac{7}{22}}$

$\Longrightarrow r = \pm\sqrt{7 \times \frac{7}{2}}$

$\Longrightarrow r = \pm\sqrt{\frac{49}{2}} = \pm\frac{7}{\sqrt{2}}$

ii. If $r^2 = l^2 + d^2$, solve for d , and find the value of 'd' if $r = 5$ and $l = 4$

$r^2 = l^2 + d^2$

$\Longrightarrow l^2 + d^2 = r^2$

$\Longrightarrow d^2 = r^2 - l^2$

$\Longrightarrow d = \pm\sqrt{r^2 - l^2}$

$\Longrightarrow d = \pm\sqrt{5^2 - 4^2}$

$\Longrightarrow d = \pm\sqrt{25 - 16}$

$\Longrightarrow d = \pm\sqrt{9} = \pm 3$

iii. If $c^2 = a^2 + b^2$, solve for b , and find the value of $'b'$ if $a = 8$ and $c = 17$

$c^2 = a^2 + b^2$

$\Longrightarrow a^2 + b^2 = c^2$

$\Longrightarrow b^2 = c^2 - a^2$

$\Longrightarrow b = \pm\sqrt{c^2 - a^2}$

$\Longrightarrow d = \pm\sqrt{17^2 - 8^2}$

$\Longrightarrow d = \pm\sqrt{289 - 64}$

$\Longrightarrow d = \pm\sqrt{225} = \pm 15$

iv. If $A = \frac{\sqrt{3}a^2}{4}$ solve for a a and find the valve of $'a'$, if $A = 16\sqrt{3}$

$$A = \frac{\sqrt{3}a^2}{4}$$

$$\Longrightarrow \frac{\sqrt{3}a^2}{4} = A$$

$$\Longrightarrow \sqrt{3}a^2 = 4A$$

$$\Longrightarrow a^2 = \frac{4A}{\sqrt{3}}$$

$$\Longrightarrow a = \pm\sqrt{\frac{4A}{\sqrt{3}}}$$

$$\Longrightarrow a = \pm\sqrt{\frac{4 \times 16\sqrt{3}}{\sqrt{3}}}$$

$$\Longrightarrow a = \pm\sqrt{64} = \pm 8$$

v. If $k = \frac{1}{2}mv^2$ solve for $'v'$ and find the value of $'v'$, if $k = 100$ and $m = 2$

$k = \frac{1}{2}mv^2$

$\Longrightarrow \frac{1}{2}mv^2 = k$

$\Longrightarrow mv^2 = 2k$

$\Longrightarrow v^2 = \frac{2k}{m}$

$\Rightarrow v = \pm\sqrt{\frac{2k}{m}}$

$\Rightarrow v = \pm\sqrt{\frac{2\times100}{2}}$

$\Rightarrow v = \pm\sqrt{100} = \pm10$

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vi. If $v^2 = u^2 + 2as$ solve for v and find the value of v if $u = 0, a = 2, s = 100$

$v^2 = u^2 + 2as$

$\Rightarrow v = \pm\sqrt{u^2 + 2as}$

$\Rightarrow v = \pm\sqrt{0^2 + 2 \times 2 \times 100}$

$\Rightarrow v = \pm\sqrt{0 + 400}$

$\Rightarrow v = \pm\sqrt{400} = \pm20$

EXERCISE 9.3

Solve the quadratic equations by factorisation method :

1. $x^2 + 15x + 50 = 0$

$x^2 + 15x + 50 = 0$

$\Rightarrow x^2 + 10x + 5x + 50 = 0$ $\quad [10 + 5 = 15, 10 \times 5 = 50]$

$\Rightarrow x(x + 10) + 5(x + 10) = 0$

$\Rightarrow (x + 10)(x + 5) = 0$

$\Rightarrow (x + 10) = 0$ or $(x + 5) = 0$

$\Rightarrow x = -10$ or $x = -5$

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2. $x^2 - 3x - 10 = 0$

$x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$ $\quad [-5 + 2 = -3, -5 \times 2 = -10]$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

$\Rightarrow (x - 5) = 0$ or $(x + 2) = 0$

$\Rightarrow x = 5$ or $x = -2$

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3. $6 - p^2 = p$

$6 - p^2 = p$

$\Rightarrow p^2 + p - 6 = 0$

$\Rightarrow p^2 + 3p - 2p - 6 = 0$ $\quad [+3 - 2 = 1, 3 \times -2 = -6]$

$\Longrightarrow p(p+3) - 2(p+3) = 0$
$\Longrightarrow (p+3)(p-2) = 0$
$\Longrightarrow (p+3) = 0 \text{ or } (p-2) = 0$
$\Longrightarrow p = -3 \text{ or } p = 2$

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4. $2x^2 + 15x - 12 = 0$

$2x^2 + 15x - 12 = 0$
$\Longrightarrow 2x^2 + 8x - 3x - 12 = 0 \quad [+8 - 3 = 5, +8 \times -3 = -24]$
$\Longrightarrow 2x(x+4) - 3(x+4) = 0$
$\Longrightarrow (x+4)(2x-3) = 0$
$\Longrightarrow (x+4) = 0 \text{ or } (2x-3) = 0$
$\Longrightarrow x = -4 \text{ or } 2x = 3 \Longrightarrow x = \frac{3}{2}$

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5. $13m = 6(m^2 + 1)$

$13m = 6(m^2 + 1)$
$\Longrightarrow 13m = 6m^2 + 6$
$\Longrightarrow 6m^2 - 13m + 6 = 0$
$\Longrightarrow 6m^2 - 9m - 4m + 6 = 0 \quad [-9 - 4 = -13, -9 \times -4 = 36]$
$\Longrightarrow 3m(2m-3) - 2(2m-3) = 0$
$\Longrightarrow (3m-2)(2m-3) = 0$
$\Longrightarrow (3m-2) = 0 \text{ or } (2m-3) = 0$
$\Longrightarrow 3m = 2 \text{ or } 2m = 3$
$\Longrightarrow m = \frac{2}{3} \text{ or } m = \frac{3}{2}$

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6. $100x^2 - 20x + 1 = 0$

$100x^2 - 20x + 1 = 0$
$\Longrightarrow 100x^2 - 10x - 10x + 1 = 0 \quad [-10 - 10 = -20, 100 \times 1 = 100]$
$\Longrightarrow 10x(10x-1) - 1(10x-1) = 0$
$\Longrightarrow (10x-1)(10x-1) = 0$
$\Longrightarrow (10x-1) = 0 \text{ or } (10x-1) = 0$
$\Longrightarrow 10x = 1 \text{ or } 10x = 1$
$\Longrightarrow x = \frac{1}{10} \text{ or } x = \frac{1}{10}$

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7. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0 \qquad [+5 + 2 = 7, 5 \times 2 = 10]$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$\Rightarrow (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$\Rightarrow (x + \sqrt{2}) = 0 \text{ or } (\sqrt{2}x + 5) = 0$

$\Rightarrow x = -\sqrt{2} \text{ or } \sqrt{2}x = -5 \Rightarrow x = \frac{-5}{\sqrt{2}}$

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8. $x^2 + 4kx + 4k^2 = 0$

$x^2 + 4kx + 4k^2 = 0$

$\Rightarrow x^2 + 2kx + 2kx + 4k^2 = 0 \qquad [+2k + 2k = 4k, 2k \times 2k = 4k^2]$

$\Rightarrow x(x + 2k) + 2k(x + 2k) = 0$

$\Rightarrow (x + 2k)(x + 2k) = 0$

$\Rightarrow (x + 2k) = 0 \text{ or } (x + 2k) = 0$

$\Rightarrow x = -2k \text{ or } x = -2k$

---

9. $m - \frac{7}{m} = 6$

$m - \frac{7}{m} = 6$

$\Rightarrow m - \frac{7}{m} = 6$

$\Rightarrow \frac{m^2 - 7}{m} = 6$

$\Rightarrow m^2 - 7 = 6m$

$\Rightarrow m^2 - 6m - 7 = 0$

$\Rightarrow m^2 - 7m + m - 7 = 0 \qquad [-7 + 1 = -6, -7 \times 1 = -7]$

$\Rightarrow m(m - 7) + 1(m - 7) = 0$

$\Rightarrow (m - 7)(m + 1) = 0$

$\Rightarrow (m - 7) = 0 \text{ or } (m + 1) = 0$

$\Rightarrow m = 7 \text{ or } m = -1$

---

10. $m - \frac{7}{m} = 6$

$x + \frac{1}{x} = 2.5$

$\Rightarrow x + \frac{1}{x} = \frac{5}{2}$

$\Rightarrow \frac{x^2 + 1}{x} = \frac{5}{2}$

$\Rightarrow 2(x^2 + 1) = 5x$

$\Rightarrow 2x^2 + 2 = 5x$

$\Longrightarrow 2x^2 - 5x + 2 = 0$

$\Longrightarrow 2x^2 - 4x - x + 2 = 0 \qquad [-4 - 1 = -5, -4 \times -1 = 4\,]$

$\Longrightarrow 2x(x - 2) - 1(x - 2) = 0$

$\Longrightarrow (x - 2)(2x - 1) = 0$

$\Longrightarrow (x - 2) = 0 \text{ or } (2x - 1) = 0$

$\Longrightarrow x = 2 \text{ or } 2x = 1 \Longrightarrow x = \frac{1}{2}$

---

11. $21y^2 = 62y + 3$

$21y^2 = 62y + 3 = 0$

$\Longrightarrow 21y^2 - 62y - 3 = 0$

$-63y + y - 3 = 0 \qquad [-63 + 1 = -62, -63 \times 1 = -63\,]$

$\Longrightarrow 21y(y - 3) + 1(y - 3) = 0$

$\Longrightarrow (y - 3)(21y + 1) = 0$

$\Longrightarrow (y - 3) = 0 \text{ or } (21y + 1) = 0$

$\Longrightarrow y = 3 \text{ or } 21y = -1 \Longrightarrow y = \frac{-1}{21}$

---

12. $0.2t^2 - 0.04t = 0.03$

$0.2t^2 - 0.04t = 0.03$

$\Rightarrow 100(0.2t^2 - 0.04t) = 100 \times 0.03$

$\Rightarrow 20t^2 - 4t = 3$

$\Rightarrow 20t^2 - 4t - 3 = 0$

$\Rightarrow 20t^2 - 10t + 6t - 3 = 0 \qquad [-10 + 6 = -4, -10 \times 6 = -60\,]$

$\Rightarrow 10t(2t - 1) + 3(2t - 1) = 0$

$\Rightarrow (2t - 1)(10t + 3) = 0$

$\Rightarrow (2t - 1) = 0 \text{ or } (10t + 3) = 0$

$\Rightarrow 2t = 1 \text{ or } 10t = -3$

$\Rightarrow t = \frac{1}{2} \text{ or } t = \frac{-3}{10}$

---

13. $4x^2 + 32x + 64 = 0$

$4x^2 + 32x + 64 = 0$

$\Longrightarrow 4(x^2 + 8x + 16) = 0$

$\Longrightarrow x^2 + 8x + 16 = 0$

$\Longrightarrow x^2 + 4x + 4x + 16 = 0 \qquad [4 + 4 = 8, 4 \times 4 = 16\,]$

$\Longrightarrow x(x + 4) + 4(x + 4) = 0$

$\Longrightarrow (x + 4)(x + 4) = 0$

$\Longrightarrow (x + 4) = 0 \text{ or } (x + 4) = 0$

$\Longrightarrow x = -4 \text{ or } x = -4$

---

14. $\sqrt{5}x^2 + 2x = 3\sqrt{5}$

$\sqrt{5}x^2 + 2x = 3\sqrt{5}$

$\Longrightarrow \sqrt{5}x^2 + 2x - 3\sqrt{5} = 0$

$\Longrightarrow \sqrt{5}x^2 + 5x - 3x - 3\sqrt{5} = 0$

$\Longrightarrow \sqrt{5}x\left(x + \sqrt{5}\right) - 3\left(x + \sqrt{5}\right) = 0 \quad [+5 - 3 = 2, +5 \times -3 = 15\,]$

$\Longrightarrow \left(x + \sqrt{5}\right)\left(\sqrt{5}x - 3\right) = 0$

$\Longrightarrow (x + \sqrt{5})(\sqrt{5}x - 3) = 0$

$\Longrightarrow \left(x + \sqrt{5}\right) = 0 \text{ or } (\sqrt{5}x - 3) = 0$

$\Longrightarrow x = -\sqrt{5} \text{ or } \sqrt{5}x = 3 \Longrightarrow x = \frac{3}{\sqrt{5}}$

---

15. $\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$

$\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$

$\Longrightarrow \frac{x^2+(x+1)^2}{x(x+1)} = \frac{34}{15}$

$\Longrightarrow \frac{x^2+x^2+2x+1}{x^2+x} = \frac{34}{15}$

$\Longrightarrow \frac{2x^2+2x+1}{x^2+x} = \frac{34}{15}$

$\Longrightarrow 34(x^2 + x) = 15(2x^2 + 2x + 1)$

$\Longrightarrow 34x^2 + 34x = 30x^2 + 30x + 15$

$\Longrightarrow 34x^2 + 34x - 30x^2 - 30x - 15 = 0$

$\Longrightarrow 4x^2 + 4x - 15 = 0$

$\Longrightarrow 4x^2 + 10x - 6x - 15 = 0 \qquad [+10 - 6 = 4, +10 \times -6 = -60]$

$\Longrightarrow 2x(2x + 5) - 3(2x + 5) = 0$

$\Longrightarrow (2x + 5)(2x - 3) = 0$

$\Longrightarrow (2x + 5) = 0 \text{ or } (2x - 3) = 0$

$\Longrightarrow 2x = -5 \text{ or } 2x = 3$

$\Longrightarrow x = \frac{-5}{2} \text{ or } x = \frac{3}{2}$

16. $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

$\Longrightarrow \frac{(x-1)(x-4)+(x-2)(x-3)}{(x-2)(x-4)} = \frac{10}{3}$

$$\Rightarrow \frac{x^2 - 5x + 4 + x^2 - 5x + 6}{x^2 - 6x + 8} = \frac{10}{3}$$
$$\Rightarrow \frac{2x^2 - 10x + 10}{x^2 - 6x + 8} = \frac{10}{3}$$
$$\Rightarrow 10(x^2 - 6x + 8) = 3(2x^2 - 10x + 10)$$
$$\Rightarrow 10x^2 - 60x + 80 = 6x^2 - 30x + 30$$
$$\Rightarrow 10x^2 - 60x + 80 - 6x^2 + 30x - 30 = 0$$
$$\Rightarrow 4x^2 - 30x + 50 = 0$$
$$\Rightarrow 2(2x^2 - 15x + 25) = 0$$
$$\Rightarrow 2x^2 - 15x + 25 = 0$$
$$\Rightarrow 2x^2 - 10x - 5x + 25 = 0$$
$$\Rightarrow 2x(x - 5) - 5(x - 5) = 0$$
$$\Rightarrow (x - 5)(2x - 5) = 0$$
$$\Rightarrow (x - 5) = 0 \text{ or} (2x - 5) = 0$$
$$\Rightarrow x = 5 \text{ or } 2x = 5 \Rightarrow x = \frac{5}{2}$$

---

17. $a^2b^2x^2 - (a^2 + b^2)x + 1 = 0$

$$a^2b^2x^2 - (a^2 + b^2)x + 1 = 0$$
$$\Rightarrow a^2b^2x^2 - a^2x - b^2x + 1 = 0$$
$$\Rightarrow a^2x(b^2x - 1) - 1(b^2x - 1) = 0$$
$$\Rightarrow (b^2x - 1)(a^2x - 1) = 0$$
$$\Rightarrow (b^2x - 1) = 0 \text{ or} (a^2x - 1) = 0$$
$$\Rightarrow b^2x = 1 \text{ or } a^2x = 1$$
$$\Rightarrow x = \frac{1}{b^2} \text{ or } x = \frac{1}{a^2}$$

---

18. $2(x + 1)^2 - 5(x + 1) = 12$

$$2(x + 1)^2 - 5(x + 1) = 12$$
$$\Rightarrow 2(x^2 + 2x + 1) - 5x - 5 = 12$$
$$\Rightarrow 2x^2 + 4x + 2 - 5x - 5 = 12$$
$$\Rightarrow 2x^2 - x - 3 = 12$$
$$\Rightarrow 2x^2 - x - 3 - 12 = 0$$
$$\Rightarrow 2x^2 - x - 15 = 0$$
$$\Rightarrow 2x^2 - 6x + 5x - 15 = 0$$
$$\Rightarrow 2x(x - 3) + 5(x - 3) = 0$$

$\Longrightarrow (x-3)(2x+5) = 0$
$\Longrightarrow (x-3) = 0 \text{ or } (2x+5) = 0$
$\Longrightarrow x = 3 \text{ or } 2x = -5 \Longrightarrow x = \frac{-5}{2}$

---

19. $(x-4)^2 + 12^2 = 15^2$

$(x-4)^2 + 12^2 = 15^2$
$\Longrightarrow x^2 - 8x + 16 + 144 = 225$
$\Longrightarrow x^2 - 8x + 160 = 225$
$\Longrightarrow x^2 - 8x + 160 - 225 = 0$
$\Longrightarrow x^2 - 8x - 65 = 0$
$\Longrightarrow x^2 - 13x + 5x - 65 = 0$
$\Longrightarrow x(x-13) + 5(x-13) = 0$
$\Longrightarrow (x-13)(x+5) = 0$
$\Longrightarrow (x-13) = 0 \text{ or } (x+5) = 0$
$\Longrightarrow x = 13 \text{ or } x = -5$

19.$2x - 3 = \sqrt{2x^2 - 2x + 21}$

$2x - 3 = \sqrt{2x^2 - 2x + 21}$
$\Longrightarrow (2x-3)^2 = \left(\sqrt{2x^2 - 2x + 21}\right)^2$
$4x^2 - 12x + 9 = 2x^2 - 2x + 21$
$\Longrightarrow 4x^2 - 12x + 9 - 2x^2 + 2x - 21 = 0$
$\Longrightarrow 2x^2 - 10x - 12 = 0$
$\Longrightarrow 2(x^2 - 5x - 6) = 0$
$\Longrightarrow x^2 - 5x - 6 = 0$
$\Longrightarrow x^2 - 6x + x - 6 = 0$
$\Longrightarrow x(x-6) + 1(x-6) = 0$
$\Longrightarrow (x-6)(x+1) = 0$
$\Longrightarrow (x-6) = 0 \text{ or } (x+1) = 0$
$\Longrightarrow x = 6 \text{ or } x = -1$

Exercise 9.4

Solve the following quadratic equations by completing the square

i. $4x^2 - 20x + 9 = 0$

$4x^2 - 20x + 9 = 0$

$\Longrightarrow 4x^2 - 20x = -9$

$\Longrightarrow 4(4x^2 - 20x) = 4 \times -9$

$\Longrightarrow 16x^2 - 80x = -36$

$\Longrightarrow 16x^2 - 80x + 100 = -36 + 100$

$\Longrightarrow (4x - 10)^2 = 64$

$\Longrightarrow 4x - 10 = \sqrt{64}$

$\Longrightarrow 4x - 10 = \pm 8$

$\Longrightarrow 4x = \pm 8 + 10$

$\Longrightarrow 4x = 8 + 10 \text{ or } 4x = -8 + 10$

$\Longrightarrow 4x = 18 \text{ or } 4x = 2$

$\Longrightarrow x = \frac{18}{4} \text{ or } x = \frac{2}{4}$

$\Longrightarrow x = \frac{9}{2} \text{ or } x = \frac{1}{2}$

$$2ab = 80x$$

$$2 \times 4x \times b = 80x$$

$$b = \frac{80x}{8x} = 10$$

$$b^2 = 100$$

---

ii. $4x^2 + x - 5 = 0$

$4x^2 + x - 5 = 0$

$4x^2 + x = 5$

$\Longrightarrow 4(4x^2 + x) = 4 \times 5$

$\Longrightarrow 16x^2 + 4x = 20$

$\Longrightarrow 16x^2 + 4x + \frac{1}{4} = 20 + \frac{1}{4}$

$\Longrightarrow \left(4x + \frac{1}{2}\right)^2 = \frac{81}{4}$

$\Longrightarrow 4x + \frac{1}{2} = \sqrt{\frac{81}{4}}$

$\Longrightarrow 4x + \frac{1}{2} = \pm \frac{9}{2}$

$\Longrightarrow 4x = \pm \frac{9}{2} - \frac{1}{2}$

$\Longrightarrow 4x = +\frac{9}{2} - \frac{1}{2} \text{ or } 4x = -\frac{9}{2} - \frac{1}{2}$

$\Longrightarrow 4x = \frac{9-1}{2} \text{ or } 4x = \frac{-9-1}{2}$

$\Longrightarrow 4x = \frac{8}{2} \text{ or} 4\,x = \frac{-10}{2}$

$$2ab = 4x$$

$$2 \times 4x \times b = 4x$$

$$b = \frac{4x}{8x} = \frac{1}{2}$$

$$b^2 = \frac{1}{4}$$

$$\Rightarrow 4x = 4 \text{ or } 4\,x = -5$$
$$\Rightarrow x = \frac{4}{4} \text{ or } x = \frac{-5}{4}$$
$$\Rightarrow x = 1 \text{ or } x = \frac{-5}{4}$$

iii. $2x^2 + 5x - 3 = 0$

$$2x^2 + 5x - 3 = 0$$
$$\Rightarrow 2x^2 + 5x = 3$$
$$\Rightarrow 2(2x^2 + 5x) = 2 \times 3$$
$$\Rightarrow 4x^2 + 10x = 6$$
$$\Rightarrow 4x^2 + 10x + \frac{25}{4} = 6 + \frac{25}{4}$$
$$\Rightarrow \left(2x + \frac{5}{2}\right)^2 = \frac{49}{4}$$
$$\Rightarrow 2x + \frac{5}{2} = \sqrt{\frac{49}{4}}$$
$$\Rightarrow 2x + \frac{5}{2} = \pm\frac{7}{2}$$
$$\Rightarrow 2x = \pm\frac{7}{2} - \frac{5}{2}$$
$$\Rightarrow 2x = +\frac{7}{2} - \frac{5}{2} \text{ or } 4x = -\frac{7}{2} - \frac{5}{2}$$
$$\Rightarrow 2x = \frac{7-5}{2} \text{ or } 2x = \frac{-7-5}{2}$$
$$\Rightarrow 2x = \frac{2}{2} \text{ or } 2\,x = \frac{-12}{2}$$
$$\Rightarrow 2x = 1 \text{ or } 2\,x = -6$$
$$\Rightarrow x = \frac{1}{2} \text{ or } x = \frac{-6}{2}$$
$$\Rightarrow x = \frac{1}{2} \text{ or } x = -3$$

$$2ab = 10x$$
$$2 \times 2x \times b = 10x$$
$$b = \frac{10x}{4x} = \frac{5}{2}$$
$$b^2 = \frac{25}{4}$$

iv. $x^2 + 16x - 9 = 0$

$$x^2 + 16x - 9 = 0$$
$$\Rightarrow x^2 + 16x = 9$$
$$\Rightarrow x^2 + 16x + 64 = 9 + 64$$

$$2ab = 16x$$
$$2 \times x \times b = 16x$$
$$b = \frac{16x}{2x} = 8$$
$$b^2 = 64$$

$\Rightarrow (x+8)^2 = 73$

$\Rightarrow x+8 = \pm\sqrt{73}$

$\Rightarrow x = -8 \pm \sqrt{73}$

$\Rightarrow x = -8+\sqrt{73}$ or $x = -8-\sqrt{73}$

---

v. $x^2 - 3x + 1 = 0$

$x^2 - 3x + 1 = 0$

$\Rightarrow x^2 - 3x = -1$

$\Rightarrow x^2 - 3x + \frac{9}{4} = -1 + \frac{9}{4}$

$\Rightarrow \left(x-\frac{3}{2}\right)^2 = \frac{-4+9}{4}$

$\Rightarrow \left(x-\frac{3}{2}\right)^2 = \frac{5}{4}$

$\Rightarrow x - \frac{3}{2} = \pm\sqrt{\frac{5}{4}}$

$\Rightarrow x = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$

$\Rightarrow x = \frac{3\pm\sqrt{5}}{2}$

$\Rightarrow x = \frac{3+\sqrt{5}}{2}$ or $x = \frac{3-\sqrt{5}}{2}$

$$2ab = 3x$$
$$2 \times x \times b = 3x$$
$$b = \frac{3x}{2x} = \frac{3}{2}$$
$$b^2 = \frac{9}{4}$$

---

vi. $t^2 + 3t = 7$

$t^2 + 3t = 7$

$\Rightarrow t^2 + 3t + \frac{9}{4} = 7 + \frac{9}{4}$

$\Rightarrow \left(t+\frac{3}{2}\right)^2 = \frac{37}{4}$

$\Rightarrow t + \frac{3}{2} = \pm\sqrt{\frac{37}{4}}$

$\Rightarrow t = -\frac{3}{2} \pm \frac{\sqrt{37}}{2}$

$\Rightarrow t = \frac{-3\pm\sqrt{37}}{2}$

$\Rightarrow t = \frac{-3+\sqrt{37}}{2}$ or $t = \frac{-3-\sqrt{37}}{2}$

$$2ab = 3t$$
$$2 \times t \times b = 3x$$
$$b = \frac{3t}{2t} = \frac{3}{2}$$
$$b^2 = \frac{9}{4}$$

vii. $3x(x-5) = 2x(x+7)$

$3x(x-5) = 2x(x+7)$

$\Rightarrow 3x^2 - 15x = 4x^2 + 14x$

$\Rightarrow 4x^2 + 14x - 3x^2 + 15x = 0$

$$2ab = 29x$$
$$2 \times x \times b = 29x$$
$$b = \frac{29x}{2x} = \frac{29}{2}$$
$$b^2 = \frac{841}{4}$$

$\Longrightarrow x^2 + 29x = 0$

$\Longrightarrow x^2 + 29x + \frac{841}{4} = \frac{841}{4}$

$\Longrightarrow \left(x + \frac{29}{2}\right)^2 = \frac{841}{4}$

$\Longrightarrow x + \frac{29}{2} = \pm\sqrt{\frac{841}{4}}$

$\Longrightarrow x + \frac{29}{2} = \pm\frac{29}{2}$

$\Longrightarrow x = -\frac{29}{2} \pm \frac{29}{2}$

$\Longrightarrow x = \frac{-29 + 29}{2}$ or $x = \frac{-29 - 29}{2}$

$\Longrightarrow x = \frac{0}{2}$ or $x = \frac{-58}{2}$

$\Longrightarrow x = 0$ or $x = -29$

---

viii. $\frac{5x+7}{x-1} = 3x + 2$

$\frac{5x+7}{x-1} = 3x + 2$

$\Longrightarrow (x - 1)(3x + 2) = 5x + 7$

$\Longrightarrow 3x^2 - x - 2 = 5x + 7$

$\Longrightarrow 3x^2 - x - 2 - 5x - 7 = 0$

$\Longrightarrow 3x^2 - 6x - 9 = 0$

$\Longrightarrow 3(x^2 - 2x - 3) = 0$

$\Longrightarrow x^2 - 2x - 3 = 0$

$\Longrightarrow x^2 - 2x = 3$

$\Longrightarrow x^2 - 2x + 1 = 3 + 1$

$\Longrightarrow (x - 1)^2 = 4$

$\Longrightarrow x - 1 = \sqrt{4}$

$\Longrightarrow x - 1 = \pm 2$

$\Longrightarrow x - 1 = 2$ or $x - 1 = -2$

$\Longrightarrow x = 2 + 1$ or $x = -2 + 1$

$\Longrightarrow x = 3$ or $x = -1$

$2ab = 2x$

$2 \times x \times b = 2x$

$b = \frac{2x}{2x} = 1$

$b^2 = 1$

ix. $a^2x^2 - 3abx + 2b^2 = 0$

$a^2x^2 - 3abx + 2b^2 = 0$

$\Longrightarrow a^2x^2 - 3abx = -2b^2$

$2ab = 3abx$

$2 \times ax \times b = 3abx$

$b = \frac{3abx}{2ax} = \frac{3b}{2}$

$b^2 = \frac{9b^2}{4}$

$$\Rightarrow a^2x^2 - 3abx + \frac{9b^2}{4} = -2b^2 + \frac{9b^2}{4}$$
$$\Rightarrow \left(ax - \frac{3b}{2}\right)^2 = \frac{-8b^2+9b^2}{4}$$
$$\Rightarrow \left(ax - \frac{3b}{2}\right)^2 = \frac{b^2}{4}$$
$$\Rightarrow ax - \frac{3b}{2} = \pm\sqrt{\frac{b^2}{4}}$$
$$\Rightarrow ax - \frac{3b}{2} = \pm\frac{b}{2}$$
$$\Rightarrow ax = \pm\frac{b}{2} + \frac{3b}{2}$$
$$\Rightarrow ax = +\frac{b}{2} + \frac{3b}{2} \text{ or } ax = -\frac{b}{2} + \frac{3b}{2}$$
$$\Rightarrow ax = \frac{+b+3b}{2} \text{ or } ax = \frac{-b+3b}{2}$$
$$\Rightarrow ax = \frac{4b}{2} \text{ or } ax = \frac{2b}{2}$$
$$\Rightarrow ax = 2b \text{ or } ax = b$$
$$\Rightarrow x = \frac{2b}{a} \text{ or } x = \frac{b}{a}$$

x. $4x^2 + 4bx - (a^2 - b^2) = 0$

$$4x^2 + 4bx - (a^2 - b^2) = 0$$
$$\Rightarrow 4x^2 + 4bx - a^2 + b^2 = 0$$
$$\Rightarrow 4x^2 + 4bx + b^2 - a^2 = 0$$
$$\Rightarrow 4x^2 + 4bx + b^2 = a^2$$
$$\Rightarrow (2x + b)^2 = a^2$$
$$\Rightarrow 2x + b = \sqrt{a^2}$$
$$\Rightarrow 2x + b = \pm a$$
$$\Rightarrow 2x = \pm a - b$$
$$\Rightarrow x = \frac{\pm a - b}{2}$$
$$\Rightarrow x = \frac{a-b}{2} \text{ or } x = \frac{-a-b}{2}$$

Exercise 9.5

Solve the following quadratic equation by using the formula method :

1. $x^2 - 4x + 2 = 0$

$x^2 - 4x + 2 = 0$

$x = x, a = 1, b = -4, c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$x = \frac{4 \pm \sqrt{8}}{2}$$

$$x = \frac{4 \pm \sqrt{4 \times 2}}{2}$$

$$x = \frac{4 \pm 2\sqrt{2}}{2}$$

$$x = \frac{2(2 \pm \sqrt{2})}{2}$$

$$x = 2 \pm \sqrt{2}$$

$x = 2 + \sqrt{2}$ or $x = 2 - \sqrt{2}$

2. $x^2 - 2x + 4 = 0$

$x^2 - 2x + 4 = 0$

$x = x, a = 1, b = -2, c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 - 16}}{2}$$

$$x = \frac{2 \pm \sqrt{-12}}{2}$$

$$x = \frac{2 \pm \sqrt{4 \times -3}}{2}$$

$$x = \frac{2 \pm 2\sqrt{-3}}{2}$$

$x = \frac{2(1 \pm \sqrt{-3})}{2}$

$x = 1 \pm \sqrt{-3}$

$x = 1 + \sqrt{-3}$ or $x = 1 - \sqrt{-3}$

3. $x^2 - 7x + 12 = 0$

$x^2 - 7x + 12 = 0$

$x = x, a = 1, b = -7, c = 12$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(12)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 - 48}}{2}$$

$$x = \frac{7 \pm \sqrt{1}}{2}$$

$$x = \frac{7 \pm 1}{2}$$

$$x = \frac{7 + 1}{2} \text{ or } x = \frac{7 - 1}{2}$$

$$x = \frac{8}{2} \text{ or } x = \frac{6}{2}$$

$x = 4$ or $x = 3$

4. $2y^2 + 6y = 3$

$2y^2 + 6y = 3$

$2y^2 + 6y - 3 = 0$

$x = y, a = 2, b = 6, c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-6 \pm \sqrt{6^2 - 4(2)(-3)}}{2(2)}$$

$$y = \frac{-6 \pm \sqrt{36 + 24}}{4}$$

$$y = \frac{-6 \pm \sqrt{60}}{4}$$

$$y = \frac{-6 \pm \sqrt{4 \times 15}}{4}$$
$$y = \frac{-6 \pm 2\sqrt{15}}{4}$$
$$y = \frac{2\left(-3 \pm \sqrt{15}\right)}{4}$$
$$y = \frac{-3 \pm \sqrt{15}}{2}$$
$$y = \frac{-3+\sqrt{15}}{2} \text{ or } y = \frac{-3-\sqrt{15}}{2}$$

---

5. $15m^2 - 11m + 2 = 0$

$15m^2 - 11m + 2 = 0$

$x = m\,, a = 15, b = -11, c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$m = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(15)(2)}}{2(15)}$$
$$m = \frac{11 \pm \sqrt{121 - 120}}{30}$$
$$m = \frac{11 \pm \sqrt{1}}{30}$$
$$m = \frac{11 \pm 1}{30}$$
$$m = \frac{11+1}{30} \text{ or } m = \frac{11-1}{30}$$
$$m = \frac{12}{30} \text{ or } m = \frac{10}{30}$$
$$m = \frac{2}{5} \text{ or } m = \frac{1}{3}$$

---

6. $8r^2 = r + 2$

$8r^2 = r + 2$

$8r^2 - r - 2 = 0$

$x = r\,, a = 8, b = -1\,, c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(8)(-2)}}{2(8)}$$

$$r = \frac{1 \pm \sqrt{1 + 64}}{16}$$

$$r = \frac{1 \pm \sqrt{65}}{16}$$

$$r = \frac{1 + \sqrt{65}}{16} \text{ or } r = \frac{1 - \sqrt{65}}{16}$$

---

7. $p = 5 - 2p^2$

$p = 5 - 2p^2$

$2p^2 + p - 5 = 0$

$x = p, a = 2, b = 1, c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$p = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-5)}}{2(2)}$$

$$p = \frac{-1 \pm \sqrt{1 + 40}}{4}$$

$$p = \frac{-1 \pm \sqrt{41}}{4}$$

$$p = \frac{-1 + \sqrt{41}}{4} \text{ or } p = \frac{-1 - \sqrt{41}}{4}$$

---

8. $(2x + 3)(3x - 2) + 2 = 0$

$(2x + 3)(3x - 2) + 2 = 0$

$6x^2 + 9x - 4x - 6 + 2 = 0$

$6x^2 + 5x - 4 = 0$

$x = x, a = 6, b = 5, c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(6)(-4)}}{2(6)}$$

$$x = \frac{-5 \pm \sqrt{25 + 96}}{12}$$

$$x = \frac{-5 \pm \sqrt{121}}{12}$$
$$x = \frac{-5 \pm 11}{4}$$
$$x = \frac{-5 + 11}{12} \text{ or } x = \frac{-5 - 11}{12}$$
$$x = \frac{6}{12} \text{ or } x = \frac{-16}{12}$$
$$x = \frac{1}{2} \text{ or } x = \frac{-4}{3}$$

---

9. $4x^2 - 4ax + (a^2 - b^2) = 0$

$$4x^2 - 4ax + (a^2 - b^2) = 0$$
$$x = x\,, a = 4, b = -4a\ \ , c = a^2 - b^2$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-4a) \pm \sqrt{(-4a)^2 - 4(4)(a^2 - b^2)}}{2(4)}$$
$$x = \frac{4a \pm \sqrt{16a^2 - 16a^2 + 16b^2}}{8}$$
$$x = \frac{4a \pm \sqrt{16b^2}}{8}$$
$$x = \frac{4a \pm 4b}{8}$$
$$x = \frac{4(a \pm b)}{8}$$
$$x = \frac{(a \pm b)}{2}$$
$$x = \frac{(a + b)}{2} \text{ or } x = \frac{(a - b)}{2}$$

---

10. $\sqrt{2x + 9} = 13 - x$

$$\sqrt{2x + 9} = 13 - x$$
$$\left(\sqrt{2x + 9}\right)^2 = (13 - x)^2$$
$$\left(\sqrt{2x + 9}\right)^2 = (13 - x)^2$$
$$2x + 9 = 169 - 26x + x^2$$
$$x^2 - 26x + 169 - 2x - 9 = 0$$

$$x^2 - 28x + 160 = 0$$
$$x = x, a = 1, b = -28, c = 160$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(1)(160)}}{2(1)}$$
$$x = \frac{28 \pm \sqrt{784 - 640}}{2}$$
$$x = \frac{28 \pm \sqrt{144}}{2}$$
$$x = \frac{28 \pm 12}{2}$$
$$x = \frac{28 + 12}{2} \text{ or } x = \frac{28 - 12}{2}$$
$$x = \frac{40}{2} \text{ or } x = \frac{16}{2}$$
$$x = 20 \text{ or } x = 8$$

---

11. $a(x^2 + 1) = x(a^2 + 1)$

$$a(x^2 + 1) = x(a^2 + 1)$$
$$ax^2 + a = a^2x + x$$
$$ax^2 + a - a^2x - x = 0$$
$$ax^2 - (a^2 + 1)x + a = 0$$
$$x = x, a = a\,, b = -(a^2 + 1), c = a$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-[-(a^2 + 1)] \pm \sqrt{[-(a^2 + 1)]^2 - 4(a)(a)}}{2a}$$
$$x = \frac{(a^2 + 1) \pm \sqrt{a^4 + 2a^2 + 1 - 4a^2}}{2a}$$
$$x = \frac{(a^2 + 1) \pm \sqrt{a^4 - 2a^2 + 1}}{2a}$$
$$x = \frac{(a^2 + 1) \pm \sqrt{(a^2 - 1)^2}}{2a}$$
$$x = \frac{(a^2 + 1) \pm (a^2 - 1)}{2a}$$

$$x = \frac{(a^2+1)+(a^2-1)}{2a} \text{ or } x = \frac{(a^2+1)-(a^2-1)}{2a}$$
$$x = \frac{a^2+1+a^2-1}{2a} \text{ or } x = \frac{a^2+1-a^2+1}{2a}$$
$$x = \frac{a^2+a^2}{2a} \text{ or } x = \frac{1+1}{2a}$$
$$x = \frac{2a^2}{2a} \text{ or } x = \frac{2}{2a}$$
$$x = a \text{ or } x = \frac{1}{a}$$

---

12. $36x^2 - 12ax + (a^2-b^2) = 0$

$$36x^2 - 12ax + (a^2-b^2) = 0$$
$$x = x\,, a = 36, b = -12a\ , c = a^2 - b^2$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-(-12a) \pm \sqrt{(-12a)^2 - 4(36)(a^2-b^2)}}{2(36)}$$
$$x = \frac{12a \pm \sqrt{144a^2 - 144(a^2-b^2)}}{72}$$
$$x = \frac{12a \pm \sqrt{144a^2 - 144a^2 + 144b^2}}{72}$$
$$x = \frac{12a \pm \sqrt{144b^2}}{72}$$
$$x = \frac{12a \pm 12b}{72}$$
$$x = \frac{12(a \pm b)}{72}$$
$$x = \frac{a \pm b}{6}$$
$$x = \frac{a+b}{6} \text{ or } x = \frac{a-b}{6}$$

---

13. $\frac{1}{x-2} + \frac{1}{x-3} + \frac{1}{x-4} = 0$

$$\frac{1}{x-2} + \frac{1}{x-3} + \frac{1}{x-4} = 0$$
$$\frac{(x-3)(x-4)+(x-2)(x-4)+(x-2)(x-3)}{(x-2)(x-3)(x-4)} = 0$$

$(x-3)(x-4)+(x-2)(x-4)+(x-2)(x-3)=0\times(x-2)(x-3)(x-4)$

$x^2-7x+12+x^2-6x+8+x^2-5x+6=0$

$3x^2-18x+26=0$

$x=x, a=3, b=-18, c=26$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-18)\pm\sqrt{(-18)^2-4(3)(26)}}{2(3)}$$

$$x=\frac{18\pm\sqrt{324-312}}{6}$$

$$x=\frac{18\pm\sqrt{12}}{6}$$

$$x=\frac{18\pm\sqrt{4\times 3}}{6}$$

$$x=\frac{18\pm 2\sqrt{3}}{6}$$

$$x=\frac{2(9\pm\sqrt{3})}{6}$$

$$x=\frac{(9\pm\sqrt{3})}{3}$$

$$x=\frac{9+\sqrt{3}}{3} \text{ or } x=\frac{9-\sqrt{3}}{3}$$

---

14. $\frac{3}{5-b}+\frac{2}{4-b}=\frac{8}{b+2}$

$$\frac{3}{5-b}+\frac{2}{4-b}=\frac{8}{b+2}$$

$$\frac{3(4-b)+2(5-b)}{(5-b)(4-b)}=\frac{8}{b+2}$$

$$\frac{12-3b+10-2b}{20-9b+b^2}=\frac{8}{b+2}$$

$$\frac{22-5b}{20-9b+b^2}=\frac{8}{b+2}$$

$8(20-9b+b^2)=(22-5b)(b+2)$

$160-72b+8b^2=22b-5b^2+44-10b$

$8b^2-72b+160=-5b^2+12b+44$

$8b^2-72b+160+5b^2-12b-44=0$

$8b^2 - 72b + 160 + 5b^2 - 12b - 44 = 0$

$13b^2 - 84b + 116 = 0$

$x = b, a = 13, b = -84, c = 116$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b = \frac{-(-84) \pm \sqrt{(-84)^2 - 4(13)(116)}}{2(13)}$

$b = \frac{84 \pm \sqrt{7056 - 6032}}{26}$

$b = \frac{84 \pm \sqrt{1024}}{26}$

$b = \frac{84 \pm 32}{26}$

$b = \frac{84 + 32}{26}$ or $b = \frac{84 - 32}{6}$

$b = \frac{116}{26}$ or $b = \frac{52}{26}$

$b = \frac{58}{13}$ or $b = 2$

Excercise 9.6

A. Discuss the nature of the following equations.

i. $y^2 - 7y + 2 = 0$

$y^2 - 7y + 2 = 0$

$a = 1, b = -7, c = 2$

$\Delta = b^2 - 4ac$

$\Delta = (-7)^2 - 4(1)(2)$

$\Delta = 49 - 8$

$\Delta = 41$

$\Delta > 0$

$\therefore$ Roots are real and distinct

ii. $x^2 - 2x + 3 = 0$

$x^2 - 2x + 3 = 0$
$a = 1, b = -2, c = 3$
$\Delta = b^2 - 4ac$
$\Delta = (-2)^2 - 4(1)(3)$
$\Delta = 4 - 12$
$\Delta = -8$
$\Delta < 0$
$\therefore$ Roots are imaginary (no real roots)

---

iii. $2n^2 + 5n - 1 = 0$
$2n^2 + 5n - 1 = 0$
$a = 2, b = 5, c = -1$
$\Delta = b^2 - 4ac$
$\Delta = 5^2 - 4(2)(-1)$
$\Delta = 25 + 8$
$\Delta = 33$
$\Delta > 0$
$\therefore$ Roots are real and distinct

---

iii. $a^2 + 4a + 4 = 0$

$a^2 + 4a + 4 = 0$
$a = 1, b = 4, c = 4$
$\Delta = b^2 - 4ac$
$\Delta = 4^2 - 4(1)(4)$
$\Delta = 16 - 16$
$\Delta = 0$
$\therefore$ Roots are real and equal

---

iv. $x^2 + 3x - 4 = 0$
$x^2 + 3x - 4 = 0$
$a = 1, b = 3, c = -4$
$\Delta = b^2 - 4ac$
$\Delta = 3^2 - 4(1)(-4)$
$\Delta = 9 + 16$
$\Delta = 25$
$\Delta > 0$

$\therefore$ Roots are real and distinct

v. $3d^2 - 2d + 1 = 0$

$3d^2 - 2d + 1 = 0$

$a = 3, b = -2, c = 1$

$\Delta = b^2 - 4ac$

$\Delta = (-2)^2 - 4(3)(1)$

$\Delta = 4 - 12$

$\Delta = -8$

$\Delta < 0$

$\therefore$ Roots are imaginary (no real roots)

B. For what positive values of $'m'$ roots of following equations are (1) equal (2) distinct (3) imaginary

i. $a^2 - ma + 1 =$

$a^2 - ma + 1 = 0$

$a = 1, b = -m, c = 1$

$\Delta = b^2 - 4ac$

$\Delta = (-m)^2 - 4(1)(1)$

$\Delta = m^2 - 4$

If roots are equal, then $\Delta = 0$

$\therefore\ m^2 - 4 = 0 \Rightarrow m^2 = 4 \Rightarrow m = \sqrt{4} = \pm 2$

If roots are distinct, then $\Delta > 0$

$\therefore\ m^2 - 4 > 0 \Rightarrow m^2 > 4 \Rightarrow m > \sqrt{4} \Rightarrow m > \pm 2$

If roots are imaginary, then $\Delta < 0$

$\therefore\ m^2 - 4 < 0 \Rightarrow m^2 < 4 \Rightarrow m < \sqrt{4} \Rightarrow m < \pm 2$

ii. $x^2 - mx + 9 = 0$

$x^2 - mx + 9 = 0$

$a = 1, b = -m, c = 9$

$\Delta = b^2 - 4ac$

$\Delta = (-m)^2 - 4(1)(9)$

$\Delta = m^2 - 36$

If roots are equal, then $\Delta = 0$

$\therefore\ m^2 - 36 = 0 \Rightarrow m^2 = 36 \Rightarrow m = \sqrt{36} = \pm 6$

If roots are distinct, then $\Delta > 0$

$\therefore\ m^2 - 36 > 0 \Rightarrow m^2 > 36 \Rightarrow m > \sqrt{36} \Rightarrow m > \pm 6$

If roots are imaginary, then $\Delta < 0$

$\therefore\ m^2 - 36 < 0 \Longrightarrow m^2 < 36 \Longrightarrow m < \sqrt{36} \Longrightarrow m < \pm 6$

---

iii. $r^2 - (m+1)r + 4 = 0$

$r^2 - (m+1)r + 4 = 0$

$a = 1, b = -(m+1)\ , c = 4$

$\Delta = b^2 - 4ac$

$\Delta = [-(m+1)]^2 - 4(1)(4)$

$\Delta = (m+1)^2 - 16$

If roots are equal, then $\Delta = 0$

$\therefore\ (m+1)^2 - 16 = 0 \Longrightarrow (m+1)^2 = 16 \Longrightarrow m + 1 = \sqrt{16} = \pm 4$

$m + 1 = \pm 4$

$m = \pm 4 - 1$

$m = +4 - 1$ or $m = -4 - 1$

$m = 3$ or $m = -5$

If roots are distinct, then $\Delta > 0$

$\therefore\ (m+1)^2 - 16 > 0 \Longrightarrow (m+1)^2 > 16 \Longrightarrow m + 1 > \sqrt{16}$

$m + 1 > \pm 4$

$m > \pm 4 - 1$

$m > +4 - 1$ or $m > -4 - 1$

$m > 3$ or $m > -5$

If roots are imaginary, then $\Delta < 0$

$\therefore\ (m+1)^2 - 16 < 0 \Longrightarrow (m+1)^2 < 16$

$\Longrightarrow m + 1 < \sqrt{16}$

$m + 1 < \pm 4$

$m < \pm 4 - 1$

$m < +4 - 1$ or $m < -4 - 1$

$m < 3$ or $m < -5$

---

iv. $mk^2 - 3k + 1 = 0$

$mk^2 - 3k + 1 = 0$

$a = m\ , b = -3, c = 1$

$\Delta = b^2 - 4ac$

$\Delta = (-3)^2 - 4(m)(1)$

$\Delta = 9 - 4m$

If roots are equal, then $\Delta = 0$

$\therefore\ 9 - 4m = 0 \Longrightarrow 4m = 9 \Longrightarrow m = \frac{9}{4}$

If roots are distinct, then $\Delta > 0$

$\therefore\ 9 - 4m > 0 \Longrightarrow 4m > 9 \Longrightarrow m > \frac{9}{4}$

If roots are imaginary, then $\Delta < 0$

$\therefore\ 9 - 4m < 0 \Longrightarrow 4m < 9 \Longrightarrow m < \frac{9}{4}$

C. Find the value of $'p'$ for which the quadratic equations have equal values.

i. $x^2 - px + 9 = 0$

$x^2 - px + 9 = 0$

$a = 1, b = -p, c = 9$

$\Delta = 0$

$\Delta = b^2 - 4ac$

$(-p)^2 - 4(1)(9) = 0$

$p^2 - 36 = 0$

$p^2 = 36$

$p = \sqrt{36} = \pm 6$

ii. $2a^2 + 3a + p$

$2a^2 + 3a + p$

$a = 2, b = 3, c = p$

$\Delta = 0$

$b^2 - 4ac = 0$

$(3)^2 - 4(2)(p) = 0$

$9 - 8p = 0$

$9 = 8p$

$p = \frac{9}{8}$

$pk^2 - 12k + 9 = 0$

$pk^2 - 12k + 9 = 0$

$a = p, b = -12, c = 9$

$\Delta = 0$

$b^2 - 4ac = 0$

$(-12)^2 - 4(p)(9) = 0$

$144 - 36p = 0$

$144 = 36p$

$p = \frac{144}{36} = 4$

---

iii. $2y^2 - py + 1 = 0$

$2y^2 - py + 1 = 0$

$a = 2, b = -p, c = 1$

$\Delta = 0$

$b^2 - 4ac = 0$

$(-p)^2 - 4(2)(1) = 0$

$p^2 - 8 = 0$

$p^2 = 8$

$p = \sqrt{8} = \sqrt{4 \times 2} = \pm 2\sqrt{2}$

---

iv. $(p+1)n^2 + 2(p+3)n + (p+8) = 0$

$(p+1)n^2 + 2(p+3)n + (p+8) = 0$

$a = p+1, b = 2(p+3), c = p+8$

$\Delta = 0$

$b^2 - 4ac = 0$

$(2(p+3))^2 - 4(p+1)(p+8) = 0$

$4(p^2 + 6p + 9) - 4(p^2 + 9p + 8) = 0$

$4p^2 + 24p + 36 - 4p^2 - 36p - 32 = 0$

$-12p + 4 = 0$

$-12p = -4$

$12p = 4$

$p = \frac{4}{12} = \frac{1}{3}$

---

v. $(3p+1)c^2 + 2(p+1)c + p = 0$

$(3p+1)c^2 + 2(p+1)c + p = 0$

$a = 3p+1, b = 2(p+1), c = p$

$\Delta = 0$

$b^2 - 4ac = 0$

$\left(2(p+1)\right)^2 - 4(3p+1)(p) = 0$

$4(p^2 + 2p + 1) - 12p^2 - 4p = 0$

$4p^2 + 8p + 4 - 12p^2 - 4p = 0$

$-8p^2 + 4p + 4 = 0$

$-8p^2 + 4p + 4 = 0$

$-4(2p^2 - p - 1) = 0$
$2p^2 - p - 1 = 0$
$2p^2 - 2p + p - 1 = 0$
$2p(p-1) + 1(p-1) = 0$
$(p-1)(2p+1) = 0$
$(p-1) = 0$ or $(2p+1) = 0$
$p = 1$ or $2p = -1 \Rightarrow p = \frac{-1}{2}$

Exercise 9.7

Find the sum and product of the roots of the quadratic equations.

1. $x^2 - 5x + 8 = 0$

$x^2 - 5x + 8 = 0$
$a = 1, b = -5, c = 8$
Sum of the roots $= \frac{-b}{a} = \frac{-(-5)}{1} = 5$
Product of the roots $= \frac{c}{a} = \frac{8}{1} = 8$

---

2. $3a^2 - 10a - 5 = 0$

$3a^2 - 10a - 5 = 0$
$a = 3, b = -10, c = -5$
Sum of the roots $= \frac{-b}{a} = \frac{-(-10)}{3} = \frac{10}{3}$
Product of the roots $= \frac{c}{a} = \frac{-5}{3}$

---

3. $8m^2 - m = 2$

$8m^2 - m = 2$
$8m^2 - m - 2 = 0$
$a = 8, b = -1, c = -2$
Sum of the roots $= \frac{-b}{a} = \frac{-(-1)}{8} = \frac{1}{8}$
Product of the roots $= \frac{c}{a} = \frac{-2}{8} = \frac{-1}{4}$

---

4. $6k^2 - 3 = 0$

$6k^2 - 3 = 0$
$a = 6, b = 0, c = -3$

Sum of the roots $= \frac{-b}{a} = \frac{0}{6} = 0$

Product of the roots $= \frac{c}{a} = \frac{-3}{6} = \frac{-1}{2}$

5. $pr^2 = r - 5$

$pr^2 = r - 5$

$pr^2 - r + 5 = 0$

$a = p, b = -1, c = 5$

Sum of the roots $= \frac{-b}{a} = \frac{-(-1)}{p} = \frac{1}{p}$

Product of the roots $= \frac{c}{a} = \frac{5}{p}$

6. $x^2 + (ab)x + (a + b) = 0$

$x^2 + (ab)x + (a + b) = 0$

$a = 1, b = ab, c = a + b$

Sum of the roots $= \frac{-b}{a} = \frac{-(ab)}{1} = ab$

Product of the roots $= \frac{c}{a} = \frac{a+b}{1} = a + b$

Exercise 9.8

A. Form the equation whose roots are

i. $3, 5$

$m = 3, n = 5$

$m + n = 3 + 5 = 8$

$mn = 3 \times 5 = 15$

Equation : $x^2 - (m + n)x + mn = 0 \Longrightarrow x^2 - 8x + 15 = 0$

ii. $6, -5$

$m = 6, n = -5$

$m + n = 6 - 5 = 1$

$mn = 6 \times -5 = -30$

Equation : $x^2 - (m + n)x + mn = 0 \Longrightarrow x^2 - x - 30 = 0$

iii. $-3, \quad \frac{3}{2}$

$m = -3, n = \frac{3}{2}$

$m + n = -3 + \frac{3}{2} = \frac{-6+3}{2} = \frac{-3}{2}$

$\mathrm{mn} = -3 \times \frac{3}{2} = \frac{-9}{2}$

Equation : $x^2 - (m+n)x + mn = 0$

$\Longrightarrow x^2 - \left(\frac{-3}{2}\right)x + \left(\frac{-9}{2}\right) = 0$

$\Longrightarrow x^2 + \frac{3}{2}x - \frac{9}{2} = 0$

$\Longrightarrow 2x^2 + 3x - 9 = 0$

---

iv. $m = \frac{2}{3}, n = \frac{3}{2}$

$m + n = \frac{2}{3} + \frac{3}{2} = \frac{4+9}{6} = \frac{13}{6}$

$mn = \frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$

Equation : $x^2 - (m+n)x + mn = 0$

$\Longrightarrow x^2 - \frac{13}{2}x + 1 = 0$

$\Longrightarrow 6x^2 - 13x + 6 = 0$

$2 + \sqrt{3}, 2 - \sqrt{3}$

$m = 2 + \sqrt{3}, n = 2 - \sqrt{3}$

$m + n = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

$mn = (2 + \sqrt{3})(2 - \sqrt{3}) = 2^2 - (\sqrt{3})^2 = 4 - 3 = 1$

Equation : $x^2 - (m+n)x + mn = 0 \Longrightarrow x^2 - 4x + 1 = 0$

---

v. $-3 + 2\sqrt{5}, -3 - 2\sqrt{5}$

$m = -3 + 2\sqrt{5}, n = -3 - 2\sqrt{5}$

$m + n = -3 + 2\sqrt{5} + -3 - 2\sqrt{5} = -6$

$mn = (-3 + 2\sqrt{5})(-3 - 2\sqrt{5}) = (-3)^2 - (2\sqrt{5})^2 = 9 - 20 = -11$

Equation : $x^2 - (m+n)x + mn = 0 \Longrightarrow x^2 - (-6)x - 11 = 0$

$\Longrightarrow x^2 + 6x - 11 = 0$

B. 1 .If $'m'$ and $'n'$ are the roots of the equation $x^2 - 6x + 2 = 0$ find the value of (i) $(m+n)mn$ (ii) $\frac{1}{m}+\frac{1}{n}$ (iii) $m^3n^2 + n^3m^2$ (iv) $\frac{1}{n}-\frac{1}{m}$

$x^2 - 6x + 2 = 0$

$a = 1, b = -6, c = 2$

Sum of the roots : $m + n = \frac{-b}{a} = \frac{-(-6)}{1} = 6$

Product of the roots : $mn = \frac{c}{a} = \frac{2}{1} = 2$

i. $(m+n)mn = 6 \times 2 = 12$

ii. $\frac{1}{m}+\frac{1}{n} = \frac{m+n}{mn} = \frac{6}{2} = 3$

iii.$m^3n^2 + n^3m^2 = m^2n^2(m+n)$

$= (mn)^2(m+n) = 2^2 \times 6 = 24$

iv. $\frac{1}{n}-\frac{1}{m} = \frac{m-n}{mn} = \frac{\pm 2\sqrt{7}}{2} = \pm\sqrt{7}$

$(m-n)^2 = m^2 + n^2 - 2mn$

$(m-n)^2 = (m+n)^2 - 2mm - 2mn$

$(m-n)^2 = (m+n)^2 - 4mn$

$(m-n)^2 = (6)^2 - 4(2)$

$(m-n)^2 = 36 - 8$

$(m-n)^2 = 28$

$m - n = \sqrt{28} = \sqrt{4 \times 7} = \pm 2\sqrt{7}$

---

2. If $'a'$ and $'b'$ are the roots of the equation $3m^2 = 6m + 5$ find the value of

(i) $\frac{a}{b}+\frac{b}{a}$ (ii) $(a+2b)(2a+b)$

$3m^2 = 6m + 5$

$3m^2 - 6m - 5 = 0$

$a = 3, b = -6, c = -5$

Sum of the roots : $a + b = \frac{-b}{a} = \frac{-(-6)}{3} = 2$

Product of the roots : $ab = \frac{c}{a} = \frac{-5}{3}$

---

i. $\frac{a}{b}+\frac{b}{a} = \frac{a^2+b^2}{ab}$

$$= \frac{(a+b)^2 - 2ab}{ab}$$

$$= \frac{(2)^2 - 2\left(\frac{-5}{3}\right)}{\frac{-5}{3}}$$

$$= \frac{4 + \frac{10}{3}}{\frac{-5}{3}}$$

$$= \frac{\frac{12+10}{3}}{\frac{-5}{3}} = \frac{\frac{22}{3}}{\frac{-5}{3}} = \frac{22}{3} \times \frac{3}{-5} = -\frac{22}{5}$$

---

ii. $(a + 2b)(2a + b)$

$= 2a^2 + 4ab + ab + 2b^2$

$= 2a^2 + 2b^2 + 5ab$

$= 2(a^2 + b^2) + 5ab$

$= 2[(a + b)^2 - 2ab] + 5ab$

$= 2\left[(2)^2 - 2\left(\frac{-5}{3}\right)\right] + 5\left(\frac{-5}{3}\right)$

$= 2\left[4 + \frac{10}{3}\right] - \frac{25}{3}$

$= 2\left[\frac{12+10}{3}\right] - \frac{25}{3}$

$= 2\left[\frac{22}{3}\right] - \frac{25}{3}$

$= \frac{44}{3} - \frac{25}{3}$

$= \frac{44-25}{3}$

$= \frac{19}{3}$

---

3. If $'p'$ and $'q'$ are the roots of the equation $2a^2 - 4a + 1 = 0$ find the value of

(i) $(p + q)^2 + 4pq$ (ii) $p^3 + q^3$

$2a^2 - 4a + 1 = 0$

$a = 2, b = -4, c = 1$

Sum of the roots : $p + q = \frac{-b}{a} = \frac{-(-4)}{2} = 2$

Product of the roots : $pq = \frac{c}{a} = \frac{1}{2}$

i. $(p + q)^2 + 4pq = 2^2 + 4\left(\frac{1}{2}\right) = 4 + 2 = 6$

ii. $p^3 + q^3 = (p + q)^3 - 3pq(p + q)$

$= 2^3 - 3\left(\frac{1}{2}\right)(2)$

$= 8 - 3 = 5$

---

4. Form a quadratic equation whose roots are $\frac{p}{q}$ and $\frac{q}{p}$

$m = \frac{p}{q}, n = \frac{q}{p}$

$m + n = \frac{p}{q} + \frac{q}{p} = \frac{p^2 + q^2}{pq}$

$mn = \frac{p}{q} \times \frac{q}{p} = \frac{pq}{pq} = 1$

Equation : $x^2 - (m+n)x + mn = 0 \Longrightarrow x^2 - \left(\frac{p^2+q^2}{pq}\right)x + 1 = 0$

$\Longrightarrow pqx^2 - (p^2 + q^2)x + pq = 0$

---

5. Find the value of $'k'$ so that the equation $x^2 + 4x + (k+2) = 0$ has one root equal to zero

$x^2 + 4x + (k+2) = 0$

$a = 1, b = 4, c = k + 2$

product of the roots : $mn = \frac{c}{a} \Longrightarrow m \times 0 = \frac{k+2}{1}$

$\Longrightarrow k + 2 = 0 \Longrightarrow k = -2$

---

6. Find the value of $'q'$ so that the equation $2x^2 - 3qx + 5q = 0$ has one root which is twice other.

$2x^2 - 3qx + 5q = 0$

$a = 2, b = -3q, c = 5q$

Sum of the roots : $m + n = \frac{-b}{a}$

$\Longrightarrow m + 2m = \frac{-(-3q)}{2}$

$\Longrightarrow 3m = \frac{3q}{2}$

$\Longrightarrow m = \frac{3q}{6} = \frac{q}{2}$

Product of the roots : $mn = \frac{c}{a}$

$\Longrightarrow m(2m) = \frac{5q}{2}$

$\Longrightarrow \left(\frac{q}{2}\right)(2 \times \frac{q}{2}) = \frac{5q}{2} \Longrightarrow q = 5$

7. Find the value of $'p'$ so that the equation $4x^2 - 8px + 9 = 0$ has roots whose difference is 4

$4x^2 - 8px + 9 = 0$

$a = 4, b = -8p, c = 9$

Sum of the roots : $m + n = \frac{-b}{a}$

$\Longrightarrow m + m + 4 = \frac{-(-8p)}{4}$ $\quad [n = m + 4]$

$\Longrightarrow 2m + 4 = 2p$

$\Longrightarrow 2(m + 2) = 2p$

$\Longrightarrow m + 2 = p$

$\Longrightarrow m = p - 2$

Product of the roots : $mn = \frac{c}{a}$

$\Longrightarrow (p - 2)(p - 2 + 4) = \frac{9}{4}$

$\Longrightarrow (p - 2)(p + 2) = \frac{9}{4}$

$\Longrightarrow p^2 - 4 = \frac{9}{4}$

$\Longrightarrow p^2 = \frac{9}{4} + 4$

$\Longrightarrow p = \sqrt{\frac{25}{4}} = \pm\frac{5}{2}$

8. If one root of the equation $x^2 + px + q = 0$ is 3 times the other prove that $3p^2 = 16q$

$x^2 + px + q = 0$

$a = 1, b = p, c = q$

Sum of the roots : $m + n = \frac{-b}{a}$

$\Longrightarrow m + 3m = \frac{-p}{1}$ $\quad [n = 3m]$

$\Longrightarrow 4m = -p$

$\Longrightarrow m = \frac{-p}{4}$

Product of the roots : $mn = \frac{c}{a}$

$\Longrightarrow m(3m) = \frac{q}{1}$

$\Longrightarrow \left(\frac{-p}{4}\right)\left(3 \times \frac{-p}{4}\right) = q$

$\Longrightarrow \frac{3p^2}{16} = q$

$\Longrightarrow 3p^2 = 16q$

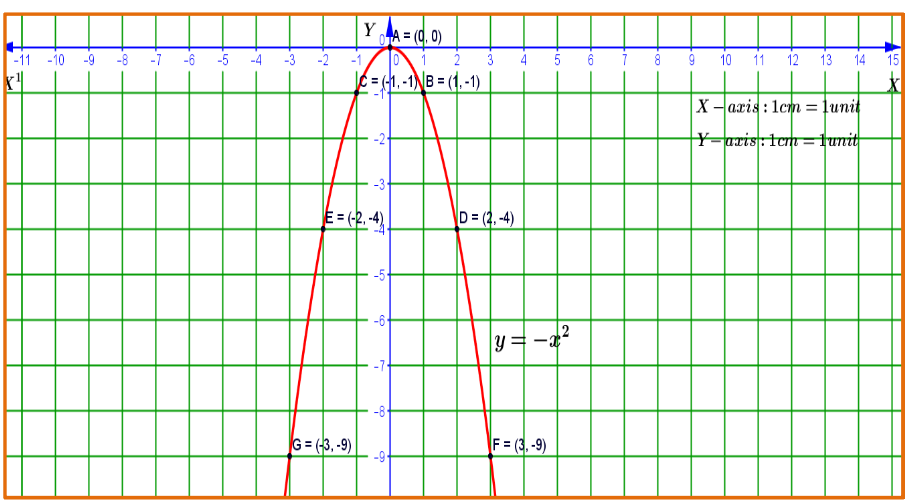
Exercise 9.9

I. Draw the graphs of the following quadratic equations :

i. $y = -x^2$

$y = -x^2$

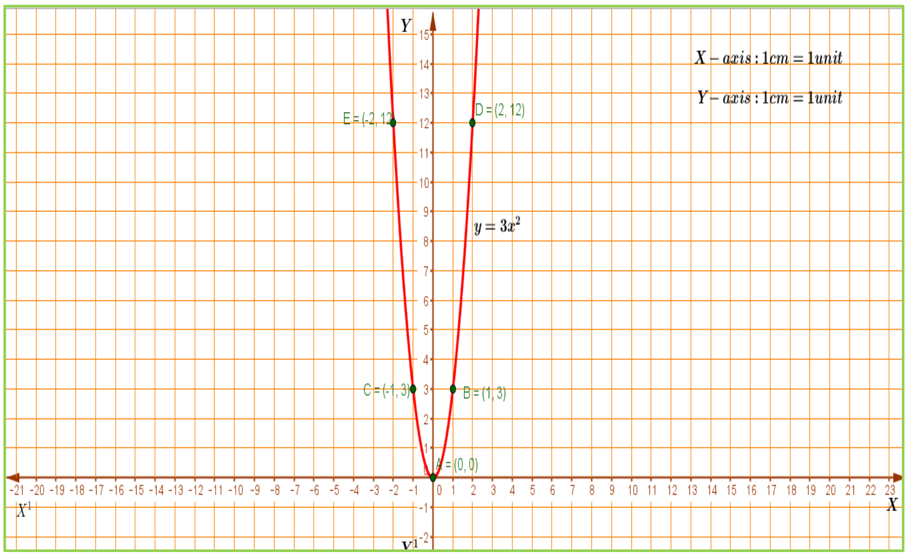
x	0	1	−1	2	−2	3	−3	4	−4
y	0	−1	−1	−4	−4	−9	−9	−16	−16



ii. $y = 3x^2$

$$y = 3x^2$$

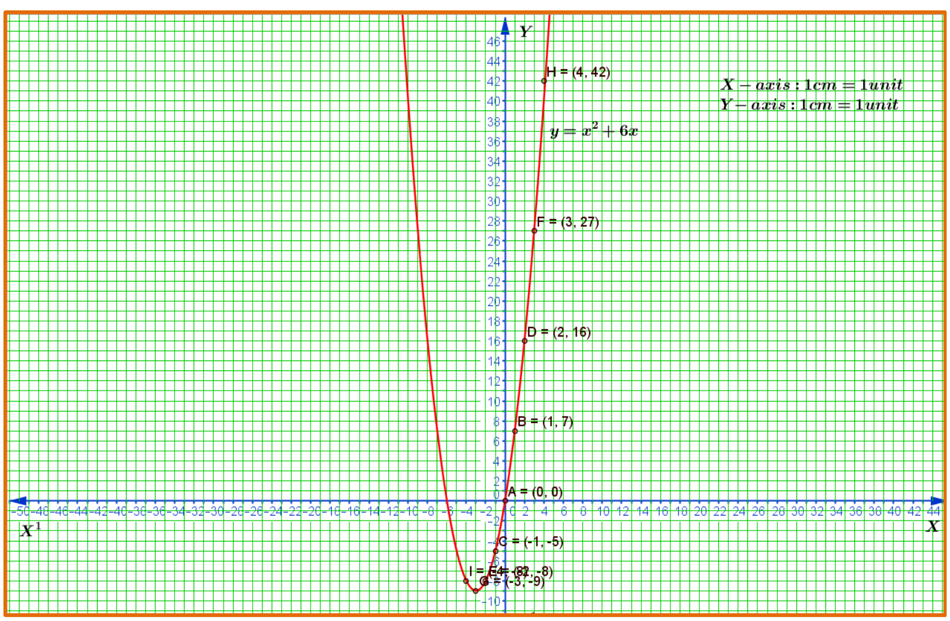
x	0	1	−1	2	−2	3	−3	4	−4
y	0	3	3	12	12	27	27	48	48



iii. $y = x^2 + 6x$

$$y = x^2 + 6x$$

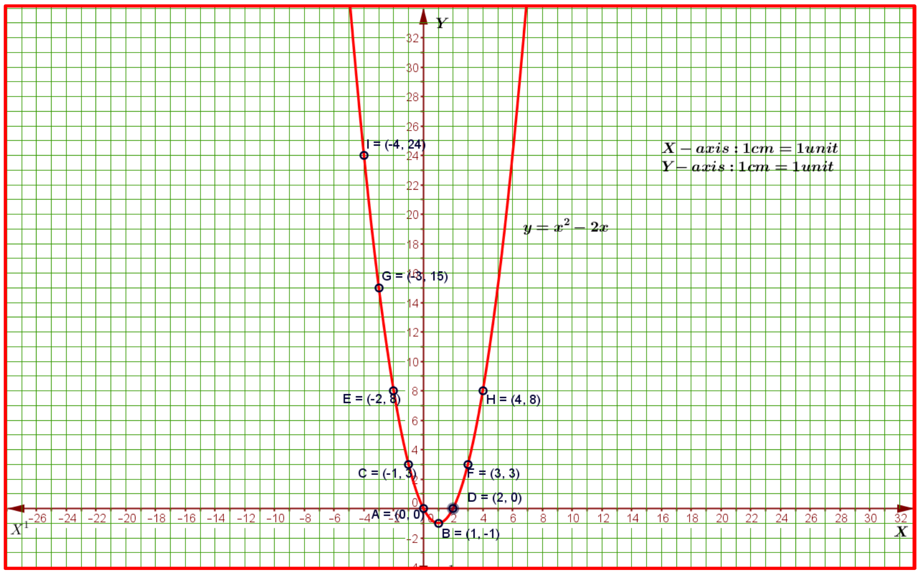
x	0	1	−1	2	−2	3	−3	4	−4
y	0	7	−5	16	−8	27	−9	42	−8



iv. $y = x^2 - 2x$

Sol : $y = x^2 - 2x$

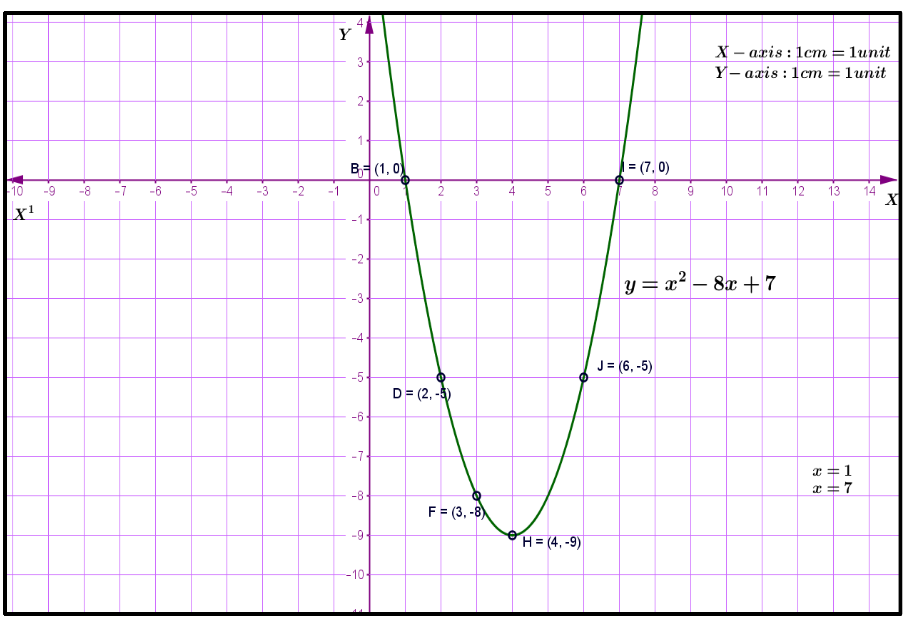
x	0	1	-1	2	-2	3	-3	4	-4
y	0	-1	3	0	8	3	15	8	24



v. $y = x^2 - 8x + 7$

Sol : $y = x^2 - 8x + 7$

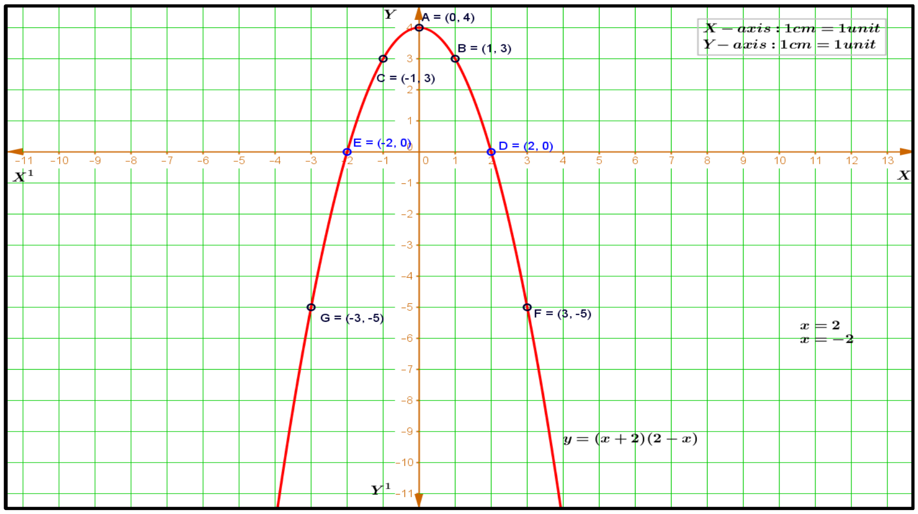
x	0	1	-1	2	-2	3	-3	6	7
y	7	0	16	-5	27	-8	40	-5	0



vi. $y = (x+2)(2-x)$

Sol : $y = (x+2)(2-x)$

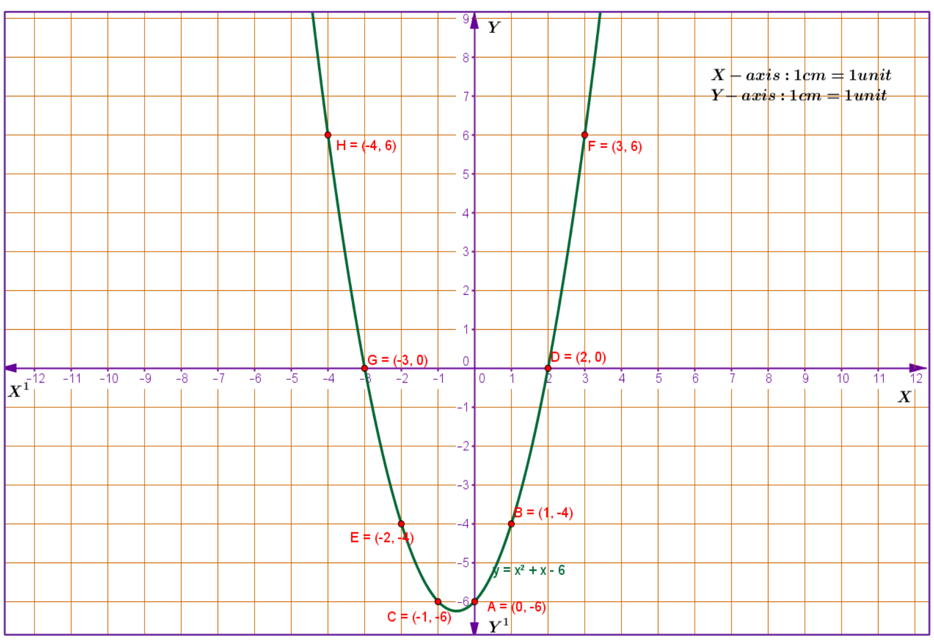
x	0	1	-1	2	-2	3	-3	4	-4
y	4	3	3	0	0	-5	-5	-12	-12



vii. $y = x^2 + x - 6$

Sol : $y = x^2 + x - 6$

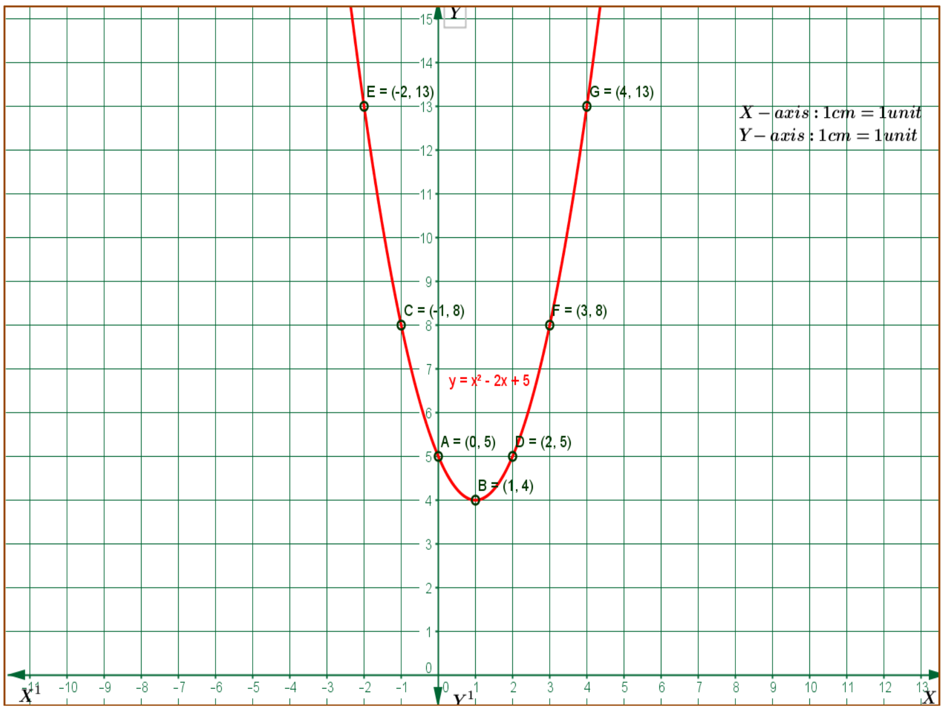
x	0	1	-1	2	-2	3	-3	4	-4
y	-6	-4	-6	0	-4	6	0	14	6



viii. $y = x^2 - 2x + 5$

Sol : $y = x^2 - 2x + 5$

x	0	1	-1	2	-2	3	-3	4	-4
y	5	4	8	5	13	8	20	13	29



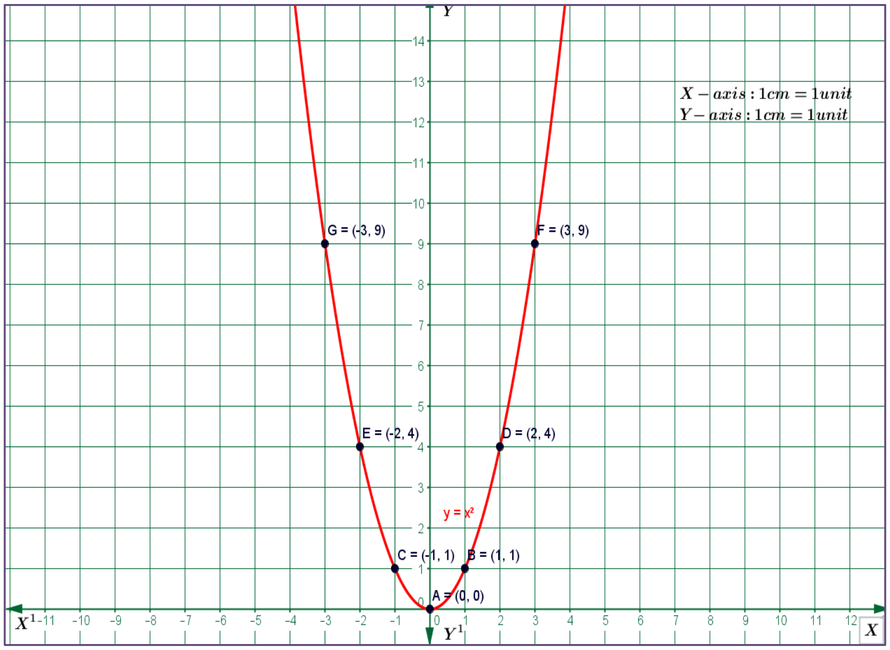
Exercise 9.10

I. Draw the graph of the following equations.

i. $y = x^2$

Sol : $y = -x^2$

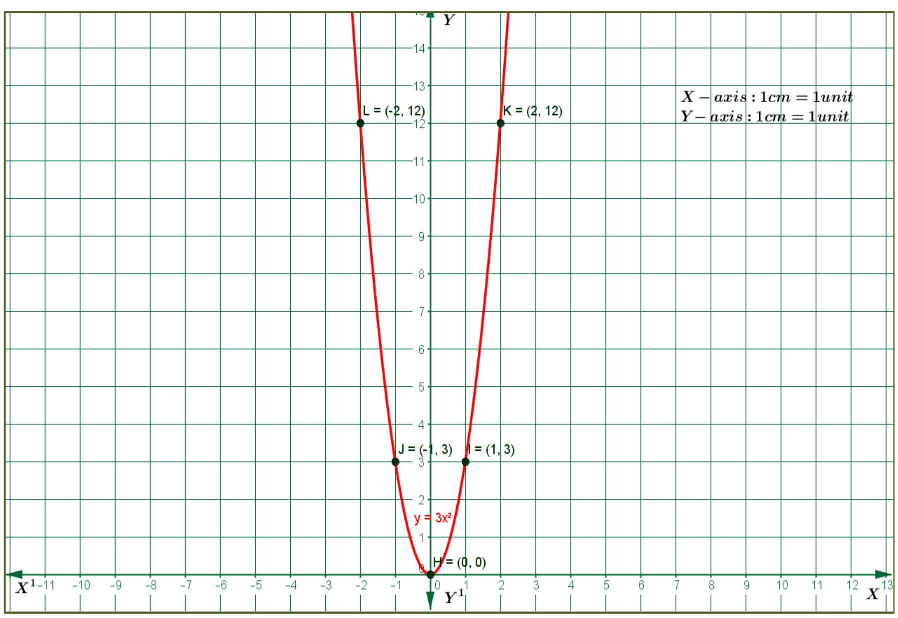
x	0	1	-1	2	-2	3	-3	4	-4
y	0	1	1	4	4	9	9	16	16



ii. $y = 3x^2$

Sol : $y = 3x^2$

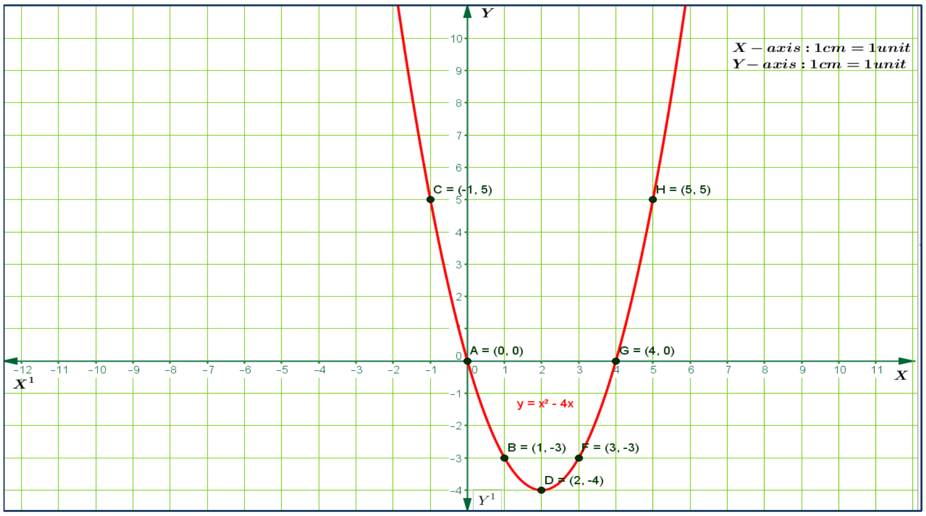
x	0	1	-1	2	-2	3	-3	4	-4
y	0	3	3	12	12	27	27	48	48



iii. $y = x^2 - 4x$

Sol : $y = x^2 - 4x$

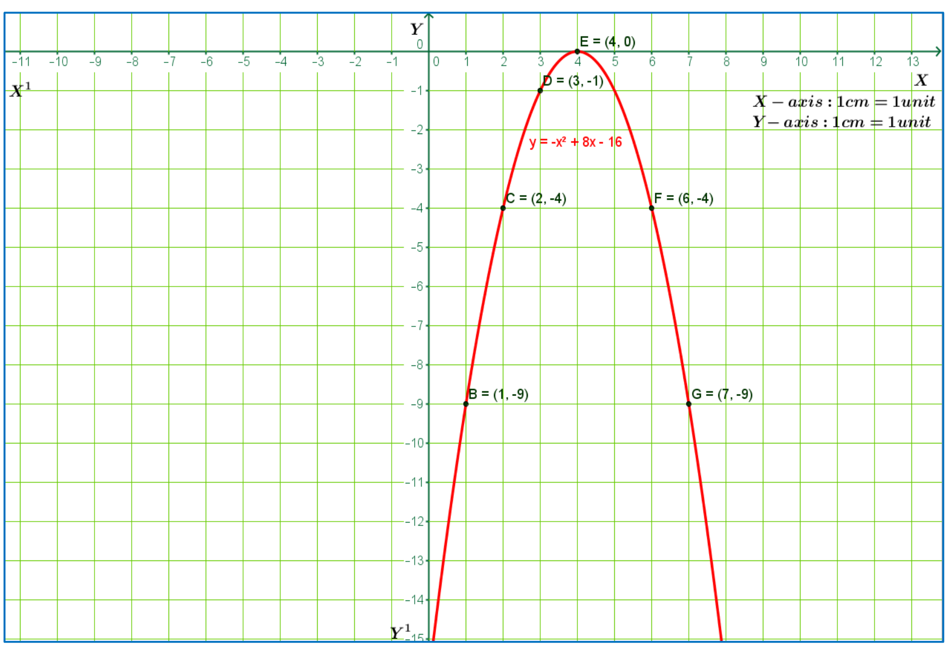
x	0	1	-1	2	-2	3	-3	4	-4
y	0	-3	5	-4	12	-3	21	0	32



iv. $y = -x^2 + 8x - 16$

Sol : $y = -x^2 + 8x - 16$

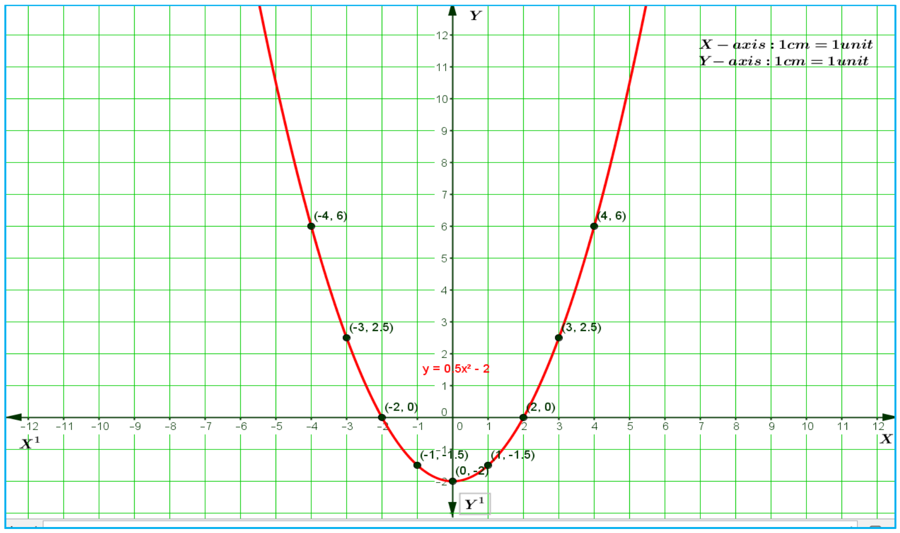
x	0	1	-1	2	-2	3	6	4	7
y	-16	-9	-25	-4	-36	-1	-4	0	-9



v. $y = \frac{1}{2}x^2 - 2$

Sol : $y = \frac{1}{2}x^2 - 2$

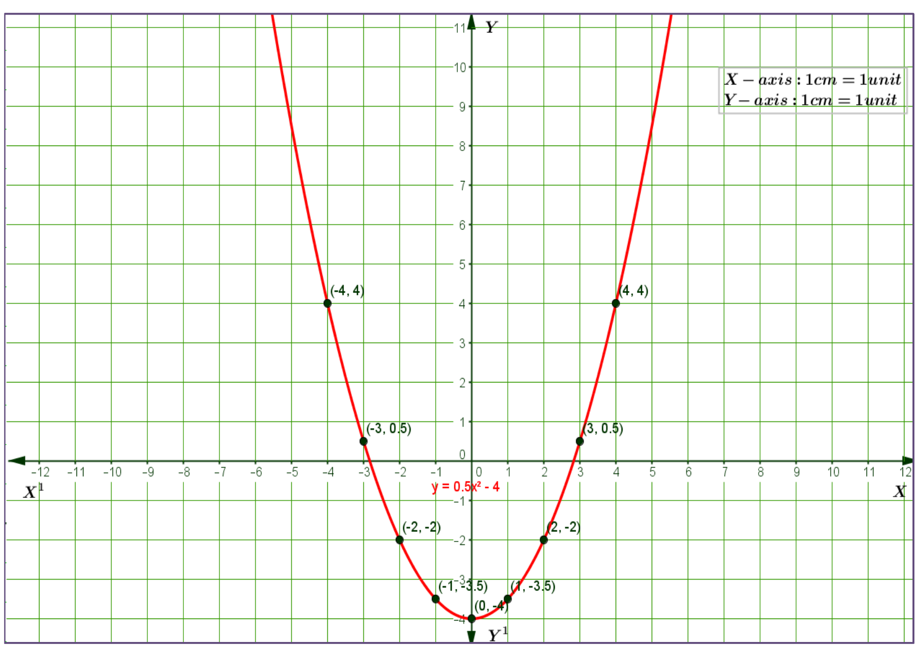
x	0	1	-1	2	-2	3	-3	4	-4
y	-2	-1.5	-1.5	0	0	2.5	2.5	6	6



vi. $y = \frac{1}{2}x^2 - 4$

Sol : $y = \frac{1}{2}x^2 - 4$

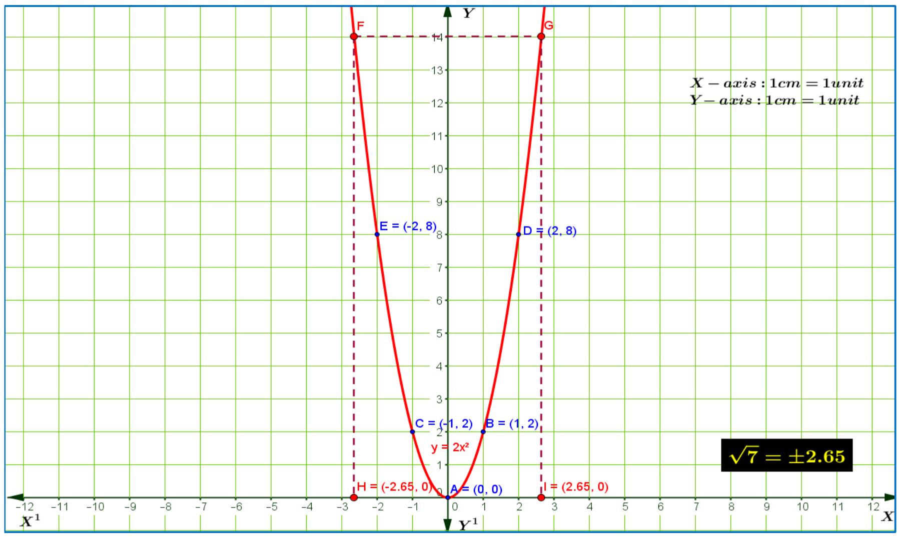
x	0	1	-1	2	-2	3	-3	4	-4
y	-4	-3.5	-3.5	-2	-2	0.5	0.5	4	4



II. 1. Draw the graph $y = x^2$ and find the value of $\sqrt{7}$

Sol : $y = 2x^2$ $\quad x = \sqrt{7} \Rightarrow y = 2\left(\sqrt{7}\right)^2 = 2 \times 7 = 14$

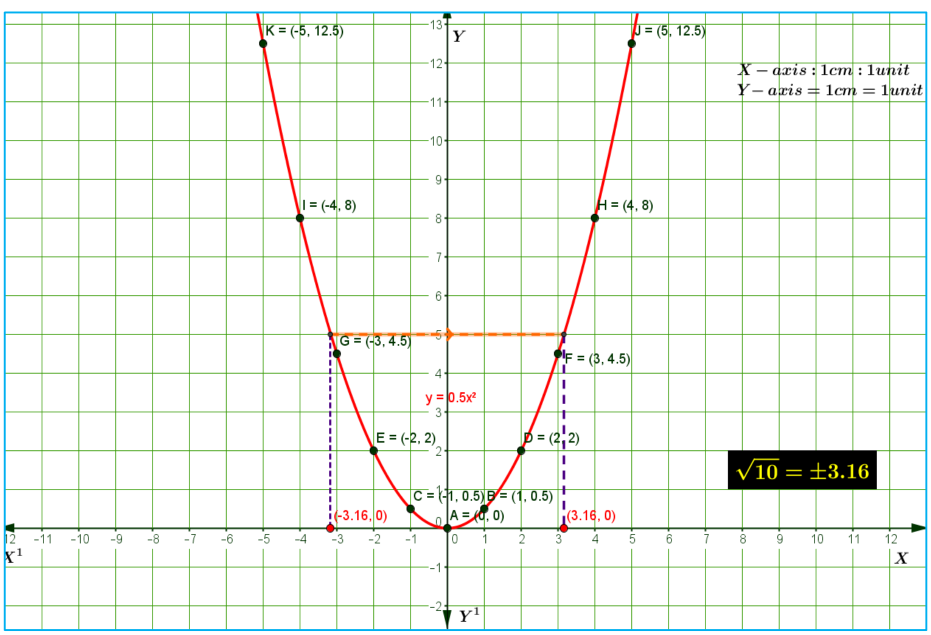
x	0	1	-1	2	-2	3	-3	4	-4
y	0	2	2	8	8	18	18	32	32



2 .Draw the graph $y = \frac{1}{2}x^2$ and find the value of $\sqrt{10}$

Sol : $y = \frac{1}{2}x^2$ $\quad x = \sqrt{10} \Rightarrow y = \frac{1}{2}\left(\sqrt{10}\right)^2 = \frac{1}{2} \times 10 = 5$

x	0	1	-1	2	-2	3	-3	4	-4
y	0	0.5	0.5	2	2	4.5	4.5	8	8



Exercise 9.11

1. Find two consecutive positive odd numbers such that the sum of their squares is equal to 130

Let the two consecutive positive odd numbers : x and x + 2

$\Longrightarrow\ x^2 + (x+2)^2 = 130$

$\Longrightarrow\ x^2 + x^2 + 4x + 4 = 130$

$\Longrightarrow\ 2x^2 + 4x + 4 - 130 = 0$

$\Longrightarrow\ 2x^2 + 4x - 126 = 0$

$\Longrightarrow\ 2(x^2 + 2x - 63) = 0$

$\Longrightarrow\ x^2 + 2x - 63 = 0$

$\Longrightarrow\ x^2 + 9x - 7x - 63 = 0$

$\Longrightarrow\ x(x+9) - 7(x+9) = 0$

$\Longrightarrow\ (x+9)(x-7) = 0$

$\Longrightarrow\ (x+9) = 0 \text{ or } (x-7) = 0$

$\Longrightarrow\ x = -9 \text{ (negative) or } x = 7$

∴ The two consecutive positive add numbers : 7,9

---

2. Find the whole number such that four times the number subtracted from three times the square of the number makes 15

Let the whole number be x

$\Longrightarrow\ 3x^2 - 4x = 15$

$\Longrightarrow\ 3x^2 - 4x - 15 = 0$

$\Longrightarrow\ 3x^2 - 9x + 5x - 15 = 0$

$\Longrightarrow\ 3x(x-3) + 5(x-3) = 0$

$\Longrightarrow\ (x-3)(3x+5) = 0$

$\Longrightarrow\ (x-3) = 0 \text{ or } (3x+5) = 0$

$\Longrightarrow\ x = 3 \text{ or } 3x = -5$

$\Longrightarrow\ x = 3 \text{ or } x = \frac{-5}{3} \text{(negative)}$

∴ whole number = 3

---

3. The sum of two natural numbers is 8. Determine the numbers, if the sum of their reciprocals is $\frac{8}{15}$

Two natural numbers : $x, 8-x$

$\Longrightarrow \frac{1}{x}+\frac{1}{8-x}=\frac{8}{15}$

$\Longrightarrow \frac{8-x+x}{x(8-x)}=\frac{8}{15}$

$\Longrightarrow \frac{8}{8x-x^2}=\frac{8}{15}$

$\Longrightarrow 8(8x-x^2)=8\times 15$

$\Longrightarrow 64x-8x^2=120$

$\Longrightarrow 8x^2-64x+120=0$

$\Longrightarrow 8(x^2-8x+15)=0$

$\Longrightarrow x^2-8x+15=0$

$\Longrightarrow x^2-5x-3x+15=0$

$\Longrightarrow x(x-5)-3(x-5)=0$

$\Longrightarrow (x-5)(x-3)=0$

$\Longrightarrow (x-5)=0$ or $(x-3)=0$

$\Longrightarrow x=5$ or $x=3$

$\therefore$ 2 natural numbers : 5,3

---

4. A two digit number is such that the product of the digits is 12. When 36 is added this number the digits interchange their places. Determine the number.

Tenth place : x Unit place : $\frac{12}{x}$ two digit number $=10x+\frac{12}{x}$

$\Longrightarrow 10x+\frac{12}{x}+36=10\times\frac{12}{x}+x$

$\Longrightarrow \frac{10x^2+12+36x}{x}=\frac{120}{x}+x$

$\Longrightarrow \frac{10x^2+12+36x}{x}=\frac{120+x^2}{x}$

$\Longrightarrow 10x^2+12+36x=120+x^2$

$\Longrightarrow 10x^2+12+36x-120-x^2=0$

$\Longrightarrow 9x^2+36x-108=0$

$\Longrightarrow 9(x^2+4x-12)=0$

$\Longrightarrow x^2+4x-12=0$

$\Longrightarrow x^2+6x-2x-12=0$

$\Longrightarrow x(x+6)-2(x+6)=0$

$\Longrightarrow\ (x+6)=0 \text{ or } (x-2)=0$
$\Longrightarrow\ x=-6 \text{ (negative) or } x=2$
$\therefore$ 2 digit number : 26

5. Find three consecutive positive integers such that the sum of the square of the first and the product of other two is 154

Sol : Three consecutive positive numbers : $x, x+1, x+2$
$\Longrightarrow x^2+(x+1)(x+2)=154$
$\Longrightarrow x^2+x^2+3x+2=154$
$\Longrightarrow 2x^2+3x+2-154=0$
$\Longrightarrow 2x^2+3x-152=0$
$\Longrightarrow 2x^2+19x-16x-152=0$
$\Longrightarrow x(2x+19)-8(2x+19)=0$
$\Longrightarrow (2x+19)(x-8)=0$
$\Longrightarrow (2x+19)=0 \text{ or } (x-8)=0$
$\Longrightarrow 2x=-19\ \Longrightarrow x=\frac{-19}{2} \text{ (negative)or } x=8$
$\therefore$ Three consecutive positive numbers : 8,9,10

6. The ages of Kavya and Karthik are 11 years and 14 years. In how many years time will the product of their ages be 304

Sol : Let the years : x
$\Longrightarrow\ (x+11)(x+14)=304$
$\Longrightarrow\ x^2+25x+154-304=0$
$\Longrightarrow\ x^2+25x-150=0$
$\Longrightarrow\ x^2+30x-5x-150=0$
$\Longrightarrow\ x(x+30)-5(x+30)=0$
$\Longrightarrow (x+30)(x-5)=0$
$\Longrightarrow (x+30)=0 \text{ or } (x-5)=0$
$\Longrightarrow x=-30 \text{ (negative)or } x=5$
$\therefore$ After 5 years the product of their ages be 304

7. The age of a man is twice the square of the age of his son. Eight years hence, the age of the man will be 4 years more than three times the age of his son. Find their present age

Age	Son	Father
Present	x	$2x^2$
After 8 years	$x+8$	$2x^2+8$

$2x^2+8=3(x+8)+4$

$\Longrightarrow 2x^2+8=3x+28$

$\Longrightarrow 2x^2+8-3x-28=0$

$\Longrightarrow 2x^2-3x-20=0$

$\Longrightarrow 2x^2-8x+5x-20=0$

$\Longrightarrow 2x(x-4)+5(x-4)=0$

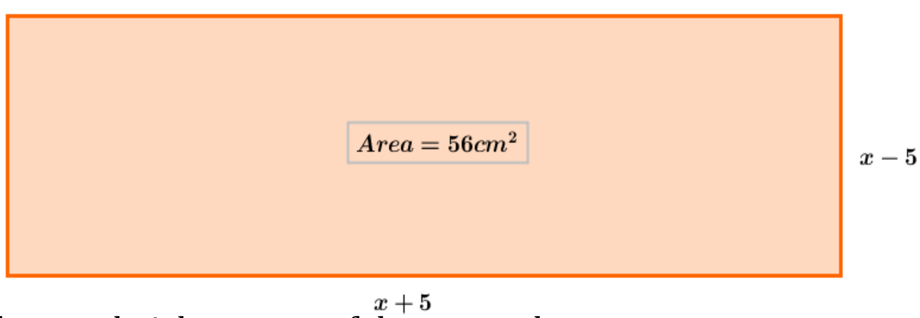
$\Longrightarrow (x-4)(2x-5)=0$

$\Longrightarrow (x-4)=0$ or $(2x-5)=0$

$\Longrightarrow x=4$ or $2x=-5 \Longrightarrow x=\frac{-5}{2}$ (negative)

$\therefore$ Age of son : 4 years , Father : 32 years

8. The area of a rectangle is $56\ \text{cm}^2$. If the measure of its base is represented by $x+5$ and the measure of its height by $x-5$, find the dimensions of the rectangle.



base $\times$ height = area of the rectangle

$\Longrightarrow (x+5)(x-5)=56$

$\Longrightarrow x^2-5^2=56$

$\Longrightarrow x^2-25=56$

$\Longrightarrow x^2=56+25$

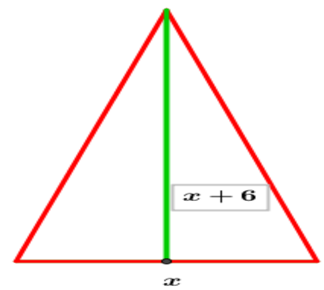
$\Longrightarrow x^2=81$

$\Longrightarrow x=\sqrt{81}\ =9$

$\therefore$ Dimensions of the rectangle : 14 cm and 4 cm

9. The altitude of a triangle is 6 cm greater than its base. If its area is 108 cm^2. Find its base.

$\frac{1}{2} \times \text{base} \times \text{height} = \text{Area of a triangle}$



$\Longrightarrow \frac{1}{2} \times x \times (x+6) = 108$

$\Longrightarrow x^2 + 6x = 216$

$\Longrightarrow x^2 + 6x - 216 = 0$

$\Longrightarrow x^2 + 18x - 12x - 216 = 0$

$\Longrightarrow x(x+18) - 12(x+18) = 0$

$\Longrightarrow (x+18)(x-12) = 0$

$\Longrightarrow (x+18) = 0 \text{ or } (x-12) = 0$

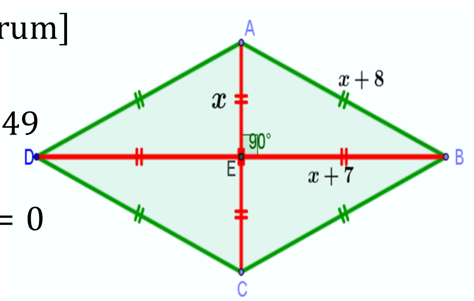
$\Longrightarrow x = -18 \text{ or } x = 12$

$\therefore$ Base $= 12$cm

10. In rhombus , the diagonals AC and BD intersect at E. If $AE = x, BE = x+7$ and $AB = x+8$, find the diagonals AC and BD

In ΔABC

$AB^2 = AE^2 + BE^2$ [$\therefore$ pytagorus theorum]



$\Longrightarrow (x+8)^2 = x^2 + (x+7)^2$

$\Longrightarrow x^2 + 16x + 64 = x^2 + x^2 + 14x + 49$

$\Longrightarrow x^2 + 16x + 64 = 2x^2 + 14x + 49$

$\Longrightarrow 2x^2 + 14x + 49 - x^2 - 16x - 64 = 0$

$\Longrightarrow x^2 - 2x - 15 = 0$

$\Longrightarrow x^2 - 5x + 3x - 15 = 0$

$\Longrightarrow x(x-5) + 3(x-5) = 0$

$\Longrightarrow x(x-5) + 3(x-5) = 0$

$\Longrightarrow (x-5)(x+3) = 0$

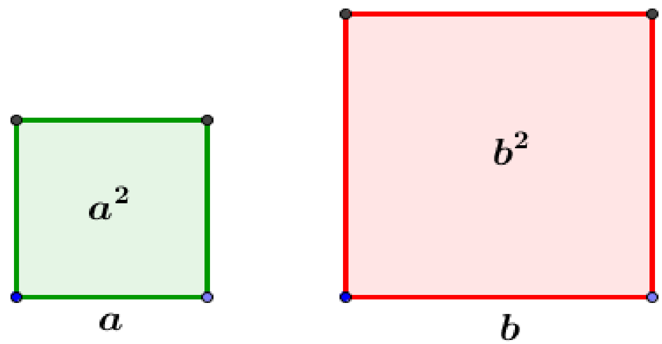
$\Longrightarrow (x-5) = 0 \text{ or } (x+3) = 0$

$\Longrightarrow x = 5 \text{ or } x = -3$ (negative)

$AE = x = 5$ cm , $BE = x + 7 = 5 + 7 = 12$ cm

$\therefore$ Diagonals $AC = 10$ cm , $BD = 24$ cm

11. If twice the area of smaller square is subtracted from the area of a larger square, the result is 14 cm^2 . However, if twice the area of the larger square is added to three times the area of the smaller square, the result is 203 cm^2. Determine the sides of the two squares.



$b^2 - 2a^2 = 14 \Rightarrow b^2 = 14 + 2a^2$

$\Longrightarrow 2b^2 + 3a^2 = 203$

$\Longrightarrow 2(14 + 2a^2) + 3a^2 = 203$

$\Longrightarrow 28 + 4a^2 + 3a^2 = 203$

$\Longrightarrow 7a^2 = 203 - 28$

$\Longrightarrow 7a^2 = 175$

$\Longrightarrow a^2 = \frac{175}{7}$

$\Longrightarrow a^2 = 25$

$\Longrightarrow a = \sqrt{25} = 5$

$\Longrightarrow b^2 = 14 + 2(5)^2$

$\Longrightarrow b^2 = 14 + 50$

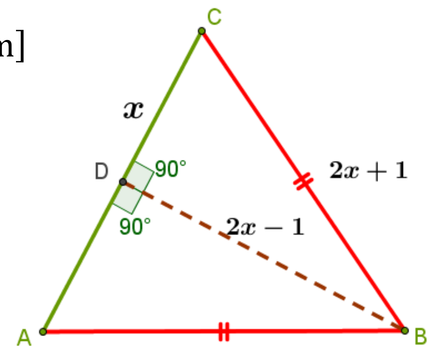
$\Longrightarrow b^2 = 64$

$\Longrightarrow b = \sqrt{64} = 8$

$\therefore$ sides of squares : 5 cm ,8 cm

12. In an isoceles triangle ABC, $AB = BC$ and BD is the altitude to base AC. If $DC = x$, $BD = 2x - 1$ and $BC = 2x + 1$, find the lengths of all three sides of the triangle.

In ΔBDc



$BC^2 = BD^2 + DC^2$ [∴ pytagorus theorum]

$\Longrightarrow (2x+1)^2 = (2x-1)^2 + x^2$

$\Longrightarrow 4x^2 + 4x + 1 = 4x^2 - 4x + 1 + x^2$

$\Longrightarrow 4x = -4x + x^2$

$\Longrightarrow 4x + 4x = x^2$

$\Longrightarrow x^2 = 8x$

$\Longrightarrow x^2 - 8x = 0$

$\Longrightarrow x(x-8) = 0$

$\Longrightarrow x = 0 \text{ or } (x-8) = 0$

$\Longrightarrow x = 0 \text{ or } x = 8$

∴ $DC = 8$ cm $\Longrightarrow AC = AD + DC = 8 + 8 = 16$ cm

∴ $AB = BC = 2x + 1 = 2(8) + 1 = 17$ cm

∴ Diagonals $AC = 10$ cm , $BD = 24$ cm

∴ Sides of triangles : 17 cm , 17 cm ,16 cm

13. A motor boat whose speed is 15 km/hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. Determine the speed of the stream.

Speed of the stream : x km/hr

Speed of the boat in still water : 15 km/hr

	speed	time
Down stream	$15 + x$	$\frac{30}{15+x}$
Up stream	$15 - x$	$\frac{30}{15-x}$

$\frac{30}{15+x} + \frac{30}{15-x} = 4$ hours 30 minutes

$$\Longrightarrow \frac{30(15+x) + 30(15-x)}{(15+x)(15-x)} = 4\ \frac{30}{60}$$

$$\Longrightarrow \frac{450+30x+450-30x}{15^2-x^2} = 4\ \frac{1}{2}$$

$\Longrightarrow \frac{900}{225-x^2} = \frac{9}{2}$
$\Longrightarrow 9(225-x^2) = 2 \times 900$
$\Longrightarrow 225 - x^2 = \frac{2\times900}{9}$
$\Longrightarrow 225 - x^2 = 200$
$\Longrightarrow x^2 = 225 - 200$
$\Longrightarrow x^2 = 25$
$\Longrightarrow x = \sqrt{25} = 5$
$\therefore$ Speed of the stream : 5 km/hr

14. A dealer sells an article for Rs 24 and gains as much percent as the cost price of the article. Find the cost price of the article.
Cost price of the article : Rs x
Selling price : Rs 24
Profit = $24 - x$
% profit = $\frac{24-x}{x} \times 100$
$x = \frac{24-x}{x} \times 100$
$x^2 = 2400 - 100x$
$x^2 + 100x - 2400 = 0$
$x^2 + 120x - 20x - 2400 = 0$
$x(x+120) - 20(x+120) = 0$
$(x+120)(x-20) = 0$
$(x+120) = 0$ or $(x-20) = 0$
$x = -120$ or $x = 20$
Cost price of the article : Rs 20

15. Nandana takes 6 days less than the number of days taken by Shobha to complete a piece of work. If both Nandana and Shobha together can complete the same work 4 days, in how many days will Shobha alone complete the work ?

	No of days	Portion of work done in one day
Shobha	x	$\frac{1}{x}$
Nandana	$x-6$	$\frac{1}{x-6}$
Together	4	$\frac{1}{4}$

$\frac{1}{x}+\frac{1}{x-6}=\frac{1}{4}$

$\frac{x-6+x}{x(x-6)}=\frac{1}{4}$

$\frac{2x-6}{x^2-6x}=\frac{1}{4}$

$x^2-6x=4(2x-6)$

$x^2-6x=8x-24$

$x^2-6x-8x+24=0$

$x^2-14x+24=0$

$x^2-12x-2x+24=0$

$x(x-12)-2(x-12)=0$

$(x-12)(x-2)=0$

$(x-12)=0$ or $(x-2)=0$

$x=12$ or $x=2$

But together they take 4 days. So one cannot complete the work in two days.

$\therefore$ Shobha takes 12 days .

16. A particle is projected from ground level so that its height above the ground after t is given by $(20t-5t^2)$m. After how many seconds is it 15 m above the ground. Can you explain briefly why are two possible answers ?

$20t-5t^2=15$

Time (seconds)	Height (metres)
t	$20t-5t^2$
?	15

$5t^2 - 20t + 15 = 0$
$5(t^2 - 4t + 3) = 0$
$t^2 - 4t + 3 = 0$
$t^2 - 3t - t + 3 = 0$
$t(t-3) - 1(t-3) = 0$
$(t-3)(t-1) = 0$
$(t-3) = 0$ or $(t-1) = 0$
$t = 3$ or $t = 1$
$20t - 5t^2 = 15 \longrightarrow$ It is a quadratic equation. it has two roots.